Section 8-1

Basics of Hypothesis Testing

HYPOTHESIS TESTING

In statistics, a **hypothesis** is a claim or statement about a property of a population.

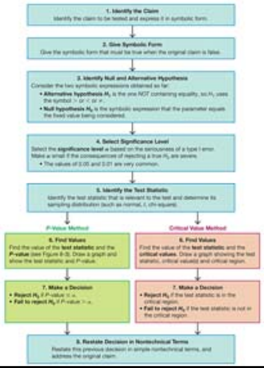
A **hypothesis test** (or **test of significance**) is a procedure for testing a claim about a property of a population.

RARE EVENT RULE

This chapter, as the last chapter, relies on the *Rare Event Rule for Inferential Statistics*.

If, under a given assumption, the probability of a particular observed event is very small and the observed event occurs *significantly less than* or *significantly greater than* what we typically expect with that assumption, we conclude that the assumption is probably not correct.

PROCUDERE FOR HYPOTHESIS TESTING



OBJECTIVES FOR SECTION 8-1

In this section, we will learn:

- How to identify the null hypothesis and alternative hypothesis from a given claim, and how to express both in symbolic form.
- How to calculate the value of the test statistic, given a claim and sample data.
- How to identify the critical value(s), given a significance level.
- How to identify the *P*-value, given the value of the test statistic.
- How to state the conclusion about a claim in simple and nontechnical terms.

NULL HYPOTHESIS

The **null hypothesis** (denoted by H_0) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is ***equal*** to some claimed value. (The term *null* is used to indicate *no* change or no effect or no difference.) Here are some examples:

H_0: $p = 0.3$ $\qquad$ H_0: $\mu = 63.6$

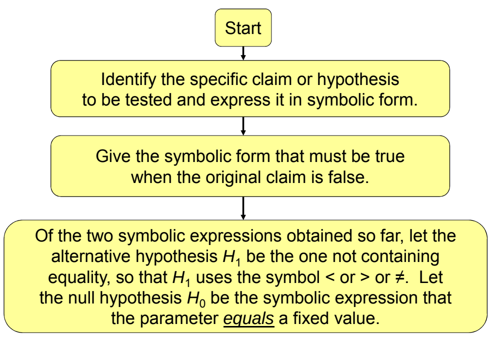
We test the null hypothesis directly in the sense that we assume it is true and reach a conclusion to either reject H_0 or fail to reject H_0.

ALTERNATIVE HYPOTHESIS

The **alternative hypothesis** (denoted by H_1 or H_a or H_A) is the statement that the parameter has a value that somehow differs from the null hypothesis. For the methods of this chapter, the symbolic form of the alternative hypothesis must use one of these symbols: $<$ or $>$ or $\neq$. For example:

Proportions:	H_1: $p > 0.3$	H_1: $p < 0.3$	H_1: $p \neq 0.3$
Means:	H_1: $\mu > 63.6$	H_1: $\mu < 63.6$	H_1: $\mu \neq 63.6$

NOTE ABOUT IDENTIFYING H_0 AND H_1



NOTE ABOUT FORMING YOUR OWN CLAIMS

If you are conducting a study and want to use a hypothesis test to ***support*** your claim, the claim must be worded so that it becomes the ***alternative hypothesis***. You can never support a claim that some parameter is *equal* to a specific value.

For example, if you want to support the claim that your IQ improvement course raises the IQ mean above 100, you must state the claim as $\mu > 100$. So, the null hypothesis is H_0: $\mu = 100$ and the alternative hypothesis is H_1: $\mu > 100$.

SIGNIFICANCE LEVEL

The **significance level** α for a hypothesis test is the probability value used as the cutoff for determining when the sample evidence constitutes *significant* evidence against the null hypothesis. By its nature, the significance level α is the probability of mistakenly rejecting the null hypothesis when it is true:

Significance level $\alpha = P(\text{rejecting } H_0 \text{ when } H_0 \text{ is true})$

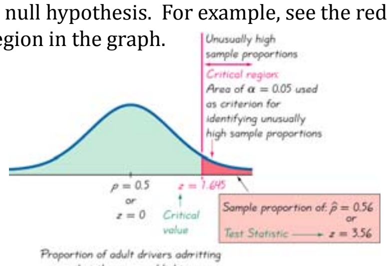
TEST STATISTIC

The **test statistic** is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

Parameter	Sampling Distribution	Requirements	Test Statistic
Proportion p	Normal (z)	$np \geq 5$ and $nq \geq 5$	$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$
Mean μ	t	σ not known and normally distributed population or σ not known and $n > 30$	$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
Mean μ	Normal (z)	σ known and normally distributed population or σ known and $n > 30$	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
St. dev. σ or variance σ^2	χ^2	Strict requirement: normally distributed population	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

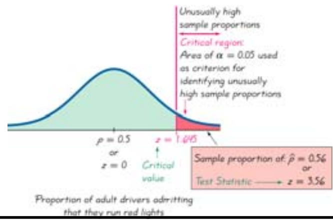
CRITICAL REGION

The **critical region** (or **rejection region**) is the set of all values of the test statistic that cause us to reject the null hypothesis. For example, see the red shaded region in the graph.



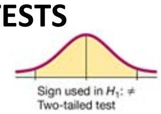
CRITICAL VALUE

The **critical value** is any value that separates the critical region from the values of the test statistic that do not lead to rejection of the null hypothesis.

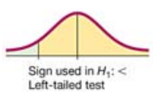


TWO-TAILED, LEFT-TAILED, AND RIGHT-TAILED TESTS

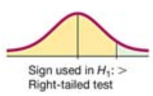
- **Two-tailed test:** The critical region is in the two extreme regions (tails) under the curve.



- **Left-tailed test:** The critical region is in the extreme left region (tail) under the curve.

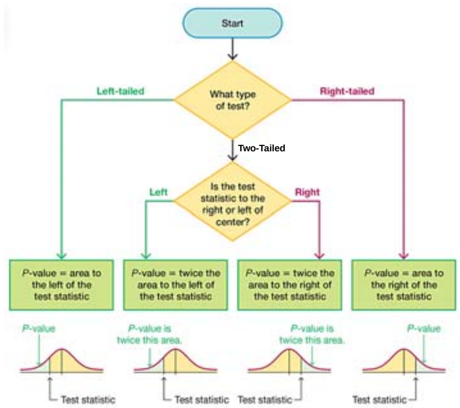


- **Right-tailed test:** The critical region is in the extreme right region (tail) under the curve.



P-VALUE

The ***P*-value** (or ***p*-value** or **probability value**) is the probability of a getting a value of the test statistic that is ***at least as extreme*** as the one representing the sample data, assuming the null hypothesis is true. The null hypothesis is rejected if the *P*-value is very small, such as 0.05 or less. *P*-values can be found by using the procedure in the flowchart on the next slide.



CONCLUSIONS

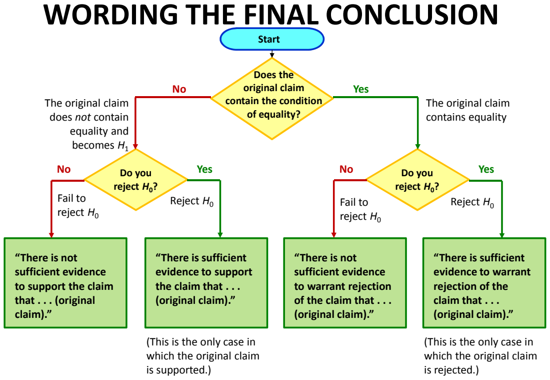
Our initial conclusion will always be one of the following:

1. Reject the null hypothesis.
2. Fail to reject the null hypothesis.

DECISION CRITERION

***P*-value Method:**	• If *P*-value ≤ α, *reject* H_0. (If *P* is low, the null must go.) • If *P*-value > α, *fail to reject* H_0.
Critical Value Method:	• If the test statistic falls within the critical region, *reject* H_0. • If the test statistic does not fall within the critical region, *fail to reject* H_0.

WORDING THE FINAL CONCLUSION



TYPE I ERROR

A **type I error** is the mistake of rejecting the null hypothesis when it is actually true.

The symbol α is used to represent the probability of a type I error.

$$\begin{aligned}\alpha &= P(\text{type I error}) \\ &= P(\text{rejecting } H_0 \text{ when } H_0 \text{ is true})\end{aligned}$$

TYPE II ERROR

A **type II error** is the mistake of failing to reject the null hypothesis when it is actually false.

The symbol β is used to represent the probability of a type II error.

$$\begin{aligned}\beta &= P(\text{type II error}) \\ &= P(\text{failing to reject } H_0 \text{ when } H_0 \text{ is false})\end{aligned}$$

TYPE I AND TYPE II ERRORS

		True State of Nature	
		Null hypothesis is true	Null hypothesis is false
Preliminary Conclusion	Reject H_0	**Type I error:** Reject a true H_0. P (type I error) $= \alpha$	Correct decision
	Fail to reject H_0	Correct decision	**Type II error:** Fail to reject a false H_0. P (type II error) $= \beta$

HINT FOR DESCRIBING TYPE I AND TYPE II ERRORS
Descriptions of a type I error and a type II error refer to the *null hypothesis* being true or false, but when wording a statement representing a type I error or a type II error, *be sure that the conclusion addresses the original claim* (which may or may not be the null hypothesis).

CONTROLLING TYPE I AND TYPE II ERRORS

- For any fixed α, an increase in the sample size n will cause a decrease in β.
- For any fixed sample size n, a decrease in α will cause an increase in β. Conversely, an increase in α will cause a decrease in β.
- To decrease both α and β, increase the sample size.