Section 7-3

Estimating a Population Standard Deviation or Variance

MAIN OBJECTIVES

1. Given sample values, estimate the population standard deviation σ or the population variance σ^2.
2. Determine the sample size required to estimate a population standard deviation or variance.

COMMENT: Estimating standard deviations is very useful in areas such a quality control in a manufacturing process. This is because manufacturers want the products to be ***consistent***.

ASSUMPTIONS

1. The sample is a simple random sample.
2. The population must have normally distributed values (even if the sample is large).

CHI-SQUARE DISTRIBUTION

To estimate a population variance we use the **chi-square distribution**.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

where n = sample size

s^2 = sample variance

σ^2 = population variance

PROPERTIES OF THE CHI-SQUARE DISTRIBUTION

1. The chi-square distribution is not symmetric, unlike the normal and Student t distributions.

   As the number of degrees of freedom increases, the distribution becomes more symmetric.

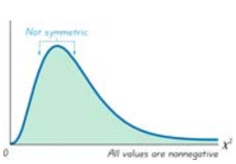

Figure 7-8 Chi-Square Distribution

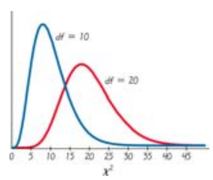

Figure 7-9 Chi-Square Distribution for *df* = 10 and *df* = 20

PROPERTIES (CONTINUED)

2. The values of chi-square can be zero or positive, but they cannot be negative.
3. The chi-square distribution is different for each number of degrees of freedom, which is $df = n - 1$ in this section. As the number increases, the chi-square distribution approaches a normal distribution.

CRITICAL VALUES

In Table A-4, each critical value of χ^2 corresponds to an area given in the top row of the table, and each area in that top row is a ***cumulative area to the right*** of the critical value.

NOTE: Since the chi-square distribution is not symmetric the left critical value (χ_L^2) and the right critical value (χ_R^2) are ***not*** just opposites of each other.

POINT ESTIMATES

- The sample variance s^2 is the best point estimate of the population variance σ^2.
- The sample standard deviation s is the best point estimate of the population standard deviation σ.

CONFIDENCE INTERVAL FOR POPULATION VARIANCE σ^2

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

right-tail critical value (χ_R^2)

left-tail critical value (χ_L^2)

CONFIDENCE INTERVAL FOR POPULATION STANDARD DEVIATION σ

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

COMMENTS

- **CAUTION**: The critical values that you look up in Table A-4 are already squared. You do ***NOT*** have to square them when you use the formulas on the previous page.
- **NOTE**: The larger critical value goes in the denominator on the left side and the smaller critical value goes in the denominator on the right.

PROCEDURE FOR CONSTRUCTING A CONFIDENCE INTERVAL FOR σ^2 OR σ

1. Verify that the required assumptions are met.
2. Using $n - 1$ degrees of freedom, refer to Table A-4 and find the critical values χ_L^2 and χ_R^2 that corresponds to the desired confidence level.
3. Evaluate the upper and lower confidence interval limits using this format of the confidence interval:

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

PROCEDURE (CONCLUDED)

4. If a confidence interval estimate of σ is desired, take the square root of the upper and lower confidence interval limits and change σ^2 to σ.
5. Round the resulting confidence level limits. If using the original set of data to construct a confidence interval, round the confidence interval limits to one more decimal place than is used for the original set of data. If using the sample standard deviation or variance, round the confidence interval limits to the same number of decimals places.

DETERMINING SAMPLE SIZE

To determine sample size, use Table 7-2 on page 338.

σ	
To be 95% confident that s is within . . .	of the value of σ, the sample size n should be at least
1%	19,205
5%	768
10%	192
20%	48
30%	21
40%	12
50%	8
To be 99% confident that s is within . . .	of the value of σ, the sample size n should be at least
1%	33,218
5%	1,336
10%	336
20%	85
30%	38
40%	22
50%	14