Section 6-4

The Central Limit Theorem

THE CENTRAL LIMIT THEOREM

<u>Central Limit Theorem</u>: For all samples of the same size n with $n > 30$, the sampling distribution of $\bar{x}$ can be approximated by a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.

THE CENTRAL LIMIT THEOREM AND THE SAMPLING DISTRIBUTION OF $\bar{x}$

<u>Given</u>:

1. The original population has mean μ and standard deviation σ.
2. Simple random samples of the same size n are selected from the population.

CLT: PRACTICAL RULES FOR REAL APPLICATIONS INVOLVING $\bar{x}$

Requirements: Population has a normal distribution or $n > 30$

Mean of all values of $\bar{x}$: $\mu_{\bar{x}} = \mu$

Std. dev. of all values of $\bar{x}$: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

z score conversion of $\bar{x}$: $z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

CLT: PRACTICAL RULES FOR REAL APPLICATIONS INVOLVING $\bar{x}$

Orginal population is _not_ normally distributed _and_ $n \leq 30$: The distribution of $\bar{x}$ cannot be approximated well by a normal distribution and the methods of this section do not apply. Use other methods, such as bootstrapping methods or nonparametric methods.

COMMENTS ON THE CENTRAL LIMIT THEOREM

The Central Limit Theorem involves ***two*** distributions.

1. The population distribution. (This is what we studied in Sections 6-1 and 6-2)
2. The distribution of sample means. (This is what we studied in the last section, Section 6-3.)

CONSIDERATIONS FOR PROBLEM SOLVING

1. **Check Requirements:** When working with the mean from a sample, verify that the normal distribution can be used by confirming the original population has a normal distribution or $n > 30$.
2. **Individual Value or Mean Value from a Sample?** Determine whether you are using a normal distribution with a *single* value x or the mean $\bar{x}$ from a sample of n values. See the following.
   - **Individual value:** When working with an *individual* value from a normally distributed population, use the methods of Section 6-2 with $z = \frac{x-\mu}{\sigma}$.
   - **Mean from a sample of values:** When working with a mean from a *sample* of n values, be sure to use the value of $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ for the standard deviation of the sample means, so use $z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$.

NOTATION FOR THE SAMPLING DISTRIBUTION OF $\bar{x}$

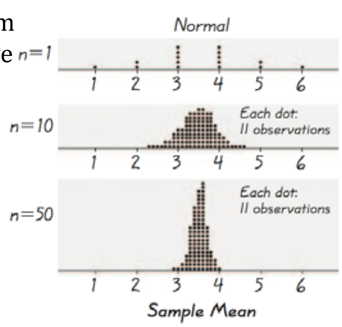
If all possible random samples of size n are selected from a population with mean μ and standard deviation σ, the mean of the sample means is denoted by $\mu_{\bar{x}}$ and the standard deviation of the all sample means is denoted by $\sigma_{\bar{x}}$. ($\sigma_{\bar{x}}$ is often called the **standard error of the mean** and is sometimes denoted as SEM.)

Mean of all values of $\bar{x}$: $\mu_{\bar{x}} = \mu$

Standard deviation of all values of $\bar{x}$: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

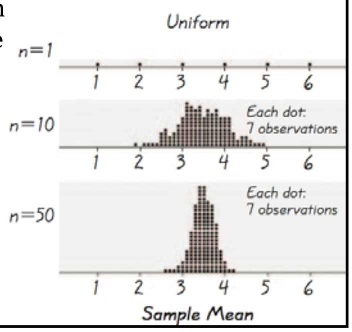
A NORMAL DISTRIBUTION

As we proceed from $n = 1$ to $n = 50$, we see that the distribution of sample means is approaching the shape of a normal distribution.



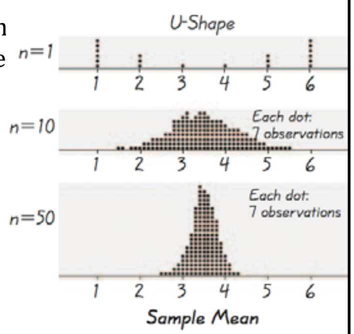
A UNIFORM DISTRIBUTION

As we proceed from $n = 1$ to $n = 50$, we see that the distribution of sample means is approaching the shape of a normal distribution.



A U-SHAPED DISTRIBUTION

As we proceed from $n = 1$ to $n = 50$, we see that the distribution of sample means is approaching the shape of a normal distribution.



As the sample size increases, the sampling distribution of sample means approaches a *normal* distribution.

CAUTIONS ABOUT THE CENTRAL LIMIT THEOREM

- When working with an ***individual*** value from a normally distributed population, use the methods of Section 6-2. Use
$$z = \frac{x - \mu}{\sigma}$$
- When working with a mean for some ***sample*** (or group) be sure to use the value of $\frac{\sigma}{\sqrt{n}}$ for the standard deviation of sample means. Use
$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

RARE EVENT RULE

If, under a given assumption, the probability of a particular observed event is exceptionally small, we conclude that the assumption is probably not correct.

EXAMPLE OF USING THE RARE EVENT RULE

The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. Assume that 25 randomly selected women are placed on a special diet just before they become pregnant.

(a) Find the probability that the lengths of pregnancy of the 25 women have a mean that is less than 260 days (assuming that the diet has no effect).

(b) If the 25 women do have a mean of less than 260 days, does it appear that the diet has an effect on the length of pregnancy, and should the medical supervisors be concerned?