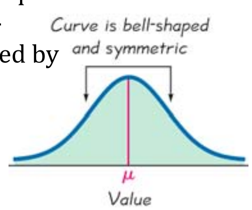
Section 6-1

The Standard Normal Distribution

NORMAL DISTRIBUTIONS

If a continuous random variable has a distribution with a graph that is symmetric and bell-shaped and can be described by the equation

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$



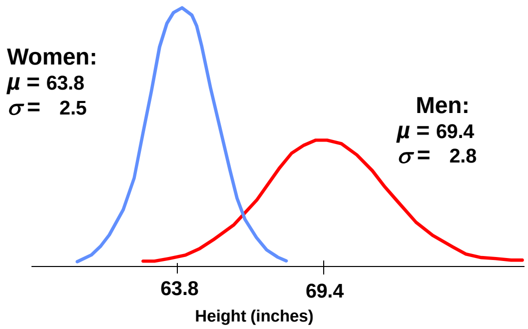
we say that it has a **normal distribution**.

REMARK

We will ***NOT*** need to use the formula on the previous slide in our work. However, it does show us one important fact about normal distributions:

Any particular normal distribution is determined by two parameters: the mean, μ, and the standard deviation, σ.

HEIGHTS OF WOMEN AND MEN



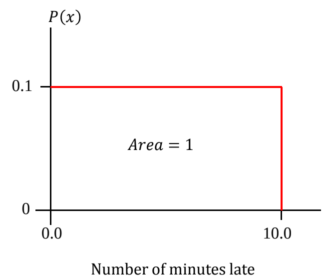
UNIFORM DISTRIBUTIONS

A continuous random variable has a **uniform distribution** if its values are spread *evenly* over the range of possibilities. The graph of a uniform distribution results in a rectangular shape.

EXAMPLE

Suppose that a friend of yours is always late. Let the random variable x represent the time from when you are suppose to meet your friend until he arrives. Your friend could be on time $(x = 0)$ or up to 10 minutes late $(x = 10)$ with all possible values equally likely.

This is an example of a uniform distribution and its graph is on the next slide.



DENSITY CURVES

A **density curve** (or **probability density function**) is a graph of a continuous probability distribution. It *must* satisfy the following properties:

1. The total area under the curve must equal 1.
2. Every point on the curve must have a vertical height that is 0 or greater. (That is, the curve cannot fall below the x-axis.)

IMPORTANT CONCEPT

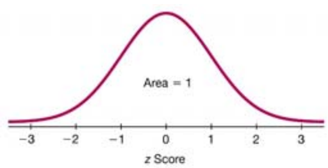
Because the total area under the density curve is equal to 1, there is a correspondence between *area* and *probability*.

EXAMPLE

Suppose that a friend of yours is always late. Let the random variable x represent the time from when you are suppose to meet your friend until he arrives. Your friend could be on time ($x = 0$) or up to 10 minutes late ($x = 10$) with all possible values equally likely. Find the probability that your friend will be more than 7 minutes late.

THE STANDARD NORMAL DISTRIBUTION

The **standard normal distribution** is a normal probability distribution that has a mean $\mu = 0$ and a standard deviation $\sigma = 1$, and the total area under the curve is equal to 1.



COMPUTING PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION

We will be computing probabilities for the standard normal distribution using:

1. Table A-2 located inside the back cover of the text, the *Formulas and Tables* insert card, and Appendix A (pp. 560-561).
2. The TI-83/84 calculator.

COMMENTS ON TABLE A-2

1. Table A-2 is designed only for the ***standard*** normal distribution
2. Table A-2 is on two pages with one page for ***negative*** z scores and the other page for ***positive*** z scores.
3. Each value in the body of the table is a ***cumulative area from the left*** up to a vertical boundary for a specific z score.

COMMENTS (CONCLUDED)

4. When working with a graph, avoid confusion between z scores and areas.

   z score: *Distance* along the horizontal scale of the standard normal distribution; refer to the leftmost column and top row of Table A-2.

   Area: *Region* under the curve; refer to the values in the *body* of the Table A-2.
5. The part of the z score denoting hundredths is found across the top row of Table A-2.

NOTATION

$P(a < z < b)$ denotes the probability that the z score is between a and b.

$P(z > a)$ denotes the probability that the z score is greater than a.

$P(z < a)$ denotes the probability that the z score is less than a.

COMPUTING PROBABILITIES USING TABLE A-2

1. Draw a bell-shaped curve corresponding to the area you are trying to find. Label the z score(s).
2. Look up the z score(s) in Table A-2.
3. Perform any necessary subtractions.

FINDING THE AREA BETWEEN TWO *z* SCORES

To find $P(a < z < b)$, the area between a and b:

1. Find the cumulative area less than a; that is, find $P(z < a)$.
2. Find the cumulative area less than b; that is, find $P(z < b)$.
3. The area between a and b is $P(a < z < b) = P(z < b) - P(z < a)$.

FINDING PROBABILITIES (AREAS) USING THE TI-83/84

To find the area between two z scores, press **2nd VARS** (for **DIST**) and select **2:normalcdf(**. Then enter the two z scores separated by a comma.

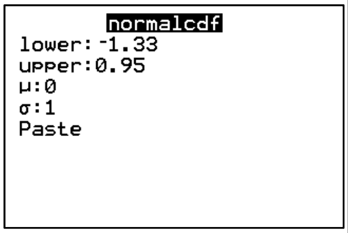
To find the area between −1.33 and 0.95, your calculator display should look like:

normalcdf(−1.33,0.95)

FINDING PROBABILITIES (AREAS) USING THE TI-84 NEW OS

To find the area between two *z* scores, press **2nd VARS** (for **DIST**) and select **2:normalcdf(**. Then enter the two *z* scores separated by a comma.

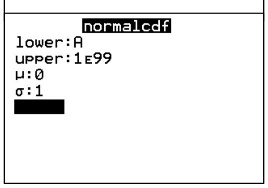
To find the area between −1.33 and 0.95, your calculator display should look like:



NOTES ON USING TI-83/84 TO COMPUTE PROBABILITIES

- To compute $P(z < a)$, use normalcdf(−1E99,a)

- To compute $P(z > a)$, use normalcdf(a,1E99)



PROCEDURE FOR FINDING A *z* SCORE FROM A KNOWN AREA USING TABLE A-2

1. Draw a bell-shaped curve and identify the region that corresponds to the given probability. If that region is not a cumulative region from the left, work instead with a known region that is cumulative from the left.
2. Using the cumulative area from the left locate the closest probability in the ***body*** of Table A-2 and identify the corresponding *z* score.

FINDING A *z* SCORE CORRESPONDING TO A KNOWN AREA USING THE TI-83/84

To find the *z* score corresponding to a known area, press **2nd VARS** (for **DIST**) and select **3:invNorm(**. Then enter the total area to the left of the value.

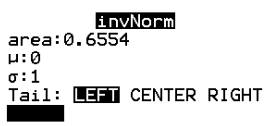
To find the *z* score corresponding to 0.6554, a cumulative area to the left, your calculator display should look like:

invNorm(.6554)

FINDING A *z* SCORE FROM AN AREA ON TI-84 NEW OS

To find the *z* score corresponding to a known area, press **2nd VARS** (for **DIST**) and select **3:invNorm(**. Then enter the total area to the left of the value.

To find the *z* score corresponding to 0.6554, a cumulative area to the left, your calculator display should look like:



CRITICAL VALUES

For the standard normal distribution, a **critical value** is a *z* score on the border line separating the *z* scores that are *significantly low* or *significantly high*.

NOTATION: The expression z_α denotes the *z* score with an area of α to its *right*. (α is the Greek lower-case letter alpha.)