Section 5-2

Binomial Probability Distributions

BINOMIAL PROBABILITY DISTRTIBUTION

A **binomial probability distribution** results from a procedure that meets all the following requirements:

1. The procedure has a ***fixed number of trials***. (A trial is a single observation.)
2. The trials must be ***independent***, meaning the outcome of any individual trial doesn't affect the probabilities in the other trials.
3. Each trial must have all outcomes classified into ***two categories***, commonly referred to as *success* or *failure*.
4. The probability of a success remains the same in all trials.

5% GUIDELINE

Treating Dependent Events as Independent: 5% Guideline for Cumbersome Calculations

When sampling without replacement and the sample size is no more than 5% of the size of the population, treat the selections as being ***independent*** (even if they are actually dependent).

NOTATION FOR BINOMIAL PROBABILITY DISTRIBUTIONS

"S" and "F" (success and failure) denote the two possible categories of all outcomes; p will denote the probability of "S" and q will denote the probability of "F". That is,

$P(S) = p$ (p = probability of success)

$P(F) = 1 - p = q$ (q = probability of failure)

NOTATION (CONCLUDED)

n denotes the fixed number of trials

x denotes the number of successes in n trials, so x can be any number between 0 and n, inclusive.

p denotes the probability of ***success*** in ***one*** of the n trials.

q denotes the probability of ***failure*** in ***one*** of the n trials.

$P(x)$ denotes the probability of getting ***exactly*** x successes among the n trials.

EXAMPLES

Identify success, failure, n, p, q, and x for the questions below. We will answer the questions later.

1. If the probability is 0.70 that a student with very high grades will get into law school, what is the probability that three of five students with very high grades will get into law school?
2. The probability is 0.60 that a person shopping in a certain market will spend \$25 or more. Find the probability that among eight persons shopping at this market, at least 6 will spend \$25 or more.

IMPORTANT CAUTIONS

- Be sure that x and p both refer to the ***same*** category being called a success.
- When sampling without replacement, the events can be treated as if they were independent if the sample size is no more than 5% of the population size. That is, $n \leq 0.05N$. (This is the 5% Guideline.)

THREE METHODS FOR FINDING BINOMIAL PROBABILITIES

1. Using the Binomial Probability Formula.
2. Using Table A-1 located in Appendix A on pages 583.
3. Using the TI-83/84 calculator.

METHOD 1: THE BINOMIAL PROBABILITY FORMULA

$$P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x} \quad \text{for } x = 0, 1, 2, \ldots, n$$

where n = number of trials
x = number of successes in n trials
p = probability of success in any one trial
q = probability of failure in any one trial
($q = 1 - p$)

This formula can also be written as

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

EXAMPLE

If the probability is 0.70 that a student with very high grades will get into law school, what is the probability that three of five students with very high grades will get into law school? Use the Binomial Probability Formula.

METHOD 2: USING TABLE A-1 IN APPENDIX A

Part of Table A-1 is shown below. With $n = 4$ and $p = 0.2$ in the binomial distribution, the probabilities of 0, 1, 2, 3, and 4 successes are 0.410, 0.410, 0.154, 0.026, and 0.002 respectively.

From Table A-1:

n	x	p 0.20
4	0	0.410
	1	0.410
	2	0.154
	3	0.026
	4	0.002

→

Binomial probability distribution for $n = 4$ and $p = 0.2$

x	$P(x)$
0	0.410
1	0.410
2	0.154
3	0.026
4	0.002

EXAMPLE

The probability is 0.60 that a person shopping in a certain market will spend $25 or more. Find the probability that among eight persons shopping at this market, at least 6 will spend $25 or more. Use Table A-1.

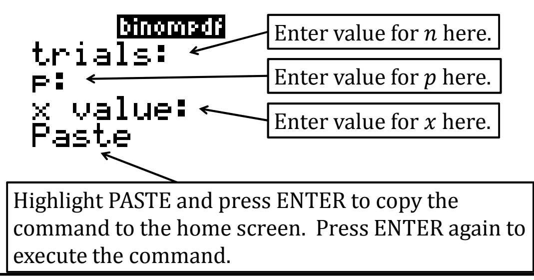
METHOD 3: USING THE TI-83/84 CALCULATOR

1. Press **2nd VARS** (to get **DISTR**)
2. Select option **0:binompdf(** on the TI-83. Use **A:binompdf(** on the TI-84.
3. Complete the entry of **binompdf(n,p,x)** with specific values for n, p, and x.
4. Press **ENTER,** and the result will be the probability of getting x successes in n trials; that is, $P(x)$.

NOTE: For more information on "binompdf(" see the Using Technology box on page 217.

TI-84 WITH NEW OPERATION SYSTEM

If using the TI-84 with the newest operating system, complete the screen as follows



EXAMPLE

Use your calculator to compute the probability of 3 successes in 10 trials if the probability of success is 0.4.

MATHEMATICAL TRANSLATIONS OF ENGLISH PHRASES

Phrase	Math Symbol
"at least" or "no less than"	$\geq$
"more than" or "greater than"	$>$
"fewer than" or "less than"	$<$
"no more than" or "at most"	$\leq$
"exactly"	$=$

CUMULATIVE BINOMIAL PROBABILITIES USING THE TI-83/84 CALCULATOR

The **cumulative binomial probability** is the sum of the individual probabilities less than or equal to a. That is,

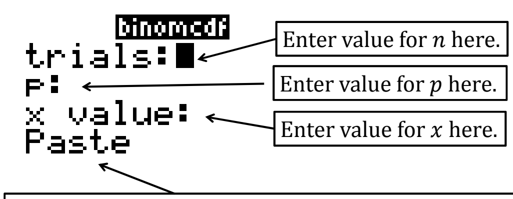
$$P(0) + P(1) + P(2) + \cdots + P(a)$$

To find the cumulative binomial probability on the calculator:

1. Press **2nd VARS** (to get **DIST**)
2. Select option **A:binomcdf(** on the TI-83. Use **B:binomcdf(** on the TI-84.
3. Complete the entry of **binomcdf(n,p,x)** with specific values for n, p, and x.
4. Press **ENTER**, and the result will be the probability of getting at most x successes in n trials, that is, $P(x \leq a)$.

CUMULATIVE BINOMIAL PROBABILITIES USING THE TI-84 WITH NEW OS

Press **2nd VARS** (to get **DIST**) and select **B:binomcdf(**.



Highlight PASTE and press ENTER to copy the command to the home screen. Press ENTER again to execute the command.

EXAMPLE

According to a Pew Research article from September 2017, 61% of young adults (ages 18-29) primarily use streaming to watch TV.

(a) In a random sample of 15 young adults, what is the probability that exactly 10 primarily use streaming to watch TV?

(b) In a random sample of 15 young adults, what is the probability that 12 or more primarily use streaming to watch TV?

(c) In a random sample of 15 young adults, what is the probability that fewer than 12 primarily use streaming to watch TV?

MEAN, VARIANCE, AND STANDARD DEVIATION

- The **mean** or **expected value** for the binomial distribution is

$$\mu = np$$

- The **variance** for the binomial distribution is

$$\sigma^2 = npq = np(1-p)$$

- The **standard deviation** for the binomial distribution is

$$\sigma = \sqrt{npq} = \sqrt{np(1-p)}$$

RANGE RULE OF THUMB

Range Rule of Thumb:

Significantly low values $\leq (\mu - 2\sigma)$

Significantly high values $\geq (\mu + 2\sigma)$

Values not significant: Between $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$

EXAMPLES

According to a Pew Research article from September 2017, 61% of young adults (ages 18-29) primarily use streaming to watch TV.

1. In a simple random sample of 300 young adults, determine the mean and standard deviation of young adults who primarily use streaming to watch TV.
2. Is the result of 195 young adults out of 300 who primarily watch TV by streaming significantly high?