Section 4-4

Counting

MULTIPLICATION COUNTING RULE

For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \cdot n$ ways.

This generalizes to more than two events.

EXAMPLES

1. How many two letter "words" can be formed if the first letter is one of the vowels a, e, i, o, u and the second letter is a consonant?
2. OVER FIFTY TYPES OF PIZZA! says the sign as you drive up. Inside you discover only the choices "onions, peppers, mushrooms, sausage, anchovies, and meatballs." Did the advertisement lie?
3. Janet has five different books that she wishes to arrange on her desk. How many different arrangements are possible?
4. Suppose Janet only wants to arrange three of her five books on her desk. How many ways can she do that?

FACTORIALS

The **factorial symbol** (**!**) denotes the product of decreasing positive whole numbers. That is,

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$$

By special definition, $0! = 1$.

FACTORIAL RULE

The number of different ***arrangements*** (order matters) of n different items when all n of them are selected is $n!$.

PERMUTATIONS AND COMBINATIONS

- **Permutations** of items are arrangements in which different sequences of the same items are counted ***separately***. (The letter arrangements ABC, ACB, BAC, BCA, CAB, and CBA are all counted ***separately*** as six different permutations.)
- **Combinations** of items are arrangements in which different sequences of the same items are counted as being the ***same***. (The letter arrangements ABC, ACB, BAC, BCA, CAB, and CBA are all considered the be the ***same*** combinations.)

PERMUTATION RULE (WHEN ITEMS ARE ALL DIFFERENT)

When n items are available and r are selected without replacement, the number of different permutations (order counts) is given by

$$_nP_r = \frac{n!}{(n-r)!}$$

PERMUTATION RULE CONDITIONS

- We must have a total of n ***different*** items available. (This rule does *not* apply if some items are ***identical*** to others.)
- We must select r of the n items without replacement.
- We must consider rearrangements of the same items to be different sequences. (The arrangement *ABC* is ***different*** from the arrangement *CBA*.)

EXAMPLE

Suppose 8 people enter an event in a swim meet. Assuming there are no ties, how many ways could the gold, silver, and bronze prizes be awarded?

PERMUTATION RULE (WHEN SOME ITEMS ARE IDENTICAL TO OTHERS)

The number of different permutations (order counts) when n items are available and all n of them are selected *without replacement*, but some of the items are identical to others is found as follows:

$$\frac{n!}{(n_1!)(n_2!)\cdots(n_k!)}$$

where n_1 are alike, n_2 are alike, … , and n_k are alike.

EXAMPLE

How many different ways can you rearrange the letters of the word "level"?

COMBINATIONS RULE

When n different items are available, but only r of them are selected *without replacement*, the number of combinations (order does not matter) is found as follows:

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

NOTE: Sometimes ${}_nC_r$ is denoted by $\binom{n}{r}$.

COMBINATIONS RULE CONDITIONS

- We must have a total of n different items available.
- We must select r of those items without replacement.
- We must consider rearrangements of the same items to be the same. (The combination ABC is the same as the combination CBA.)

EXAMPLES

1. From a group of 30 employees, 3 are to be selected to be on a special committee. In how many different ways can the employees be selected?
2. If you play the New York regional lottery where six winning numbers are drawn from 1, 2, 3, . . . , 31, what is the probability that you are a winner?
3. The Mega Millions Lottery is run in 44 states. Winning the jackpot requires that you select the correct five numbers between 1 and 70 and, in a separate drawing, you must also select the correct single number between 1 and 25. Find the probability of winning the jackpot.

EXAMPLE

Suppose you are dealt two cards from a well-shuffled deck. What is the probability of being dealt an “ace” and a “heart”?

PERMUTATIONS VERSUS COMBINATIONS

When different orderings of the same items are to be ***counted separately***, we have a ***permutation*** problem, but when different orderings are ***NOT to be counted*** separately, we have a ***combination*** problem.