Section 4-2

The Addition Rule and Multiplication Rule

COMPOUND EVENT

A **compound event** is any event combining two or more simple events.

NOTATION FOR THE ADDITION RULE

$$P(A \text{ or } B) = P(\text{in a single trial, event } A \text{ occurs or event } B \text{ occurs or they both occur})$$

INTUITIVE ADDITION RULE

To find $P(A \text{ or } B)$, add the number of ways event A can occur and the number of ways event B can occur, but ***add in such a way that every outcome is counted only once***.
$P(A \text{ or } B)$ is equal to that sum, divided by the total number of outcomes in the sample space.

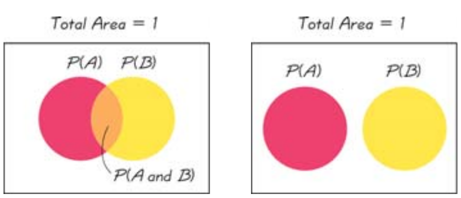
FORMAL ADDITION RULE

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

where $P(A \text{ and } B)$ denotes the probability that A and B both occur at the same time as an outcome in a trial of a procedure.

DISJOINT EVENTS

Events A and B are **disjoint** (or **mutually exclusive**) if they cannot both occur together. (That is, disjoint events do not overlap.)



EXAMPLES

Determine whether the two events are disjoint for a single trial.

1. Arriving late for your next statistics class.
   Arriving early for your next statistics class.

2. Asking for a date through a Twitter post.
   Asking for a date in French, the romance language.

3. Randomly selecting a drug screening result and getting one that is a false positive.
   Randomly selecting a drug screening result and getting one from someone who uses drugs.

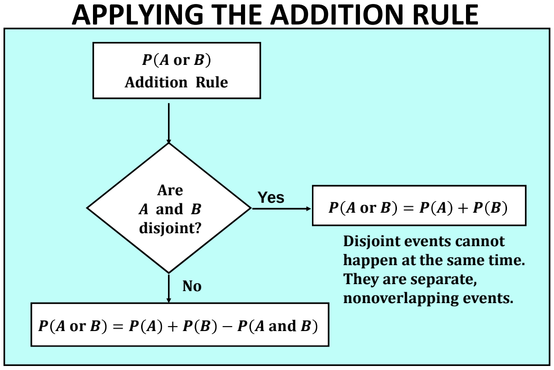
4. Randomly selecting a drug screening result and getting one that is a false positive.
   Randomly selecting a drug screening result and getting one that is a false negative.

OBSERVATIONS ON DISJOINT EVENTS

- If two events, A and B, are disjoint, then $P(A \text{ and } B) = 0$.
- If events A and B are disjoint, then

$$P(A \text{ or } B) = P(A) + P(B).$$

APPLYING THE ADDITION RULE



EXAMPLE

The data in the chart below represent the marital status of males and females 18 years or older in the US in 2010. Use it to answer the questions on the next slide.

(Source: US Census Bureau)	**Males (in millions)**	**Females (in millions)**	**Totals (in millions)**
Never Married	**33.7**	**27.8**	**61.5**
Married	**64.4**	**65.1**	**129.5**
Widowed	**3.0**	**11.4**	**14.4**
Divorced	**10.0**	**13.7**	**23.7**
Totals (in millions)	**111.1**	**118.0**	**229.1**

EXAMPLE (CONCLUDED)

1. Determine the probability that a randomly selected United States resident 18 years or older is male.
2. Determine the probability that a randomly selected United States resident 18 years or older is widowed.
3. Determine the probability that a randomly selected United States resident 18 years or older is widowed or divorced.
4. Determine the probability that a randomly selected United States resident 18 years or older is male or widowed.

CAUTION

CAUTION: Errors made when applying the addition rule often involve double counting; that is, events that are not disjoint are treated as if they were. One indication of such an error is a total probability that exceeds 1; however, errors involving the addition rule do not always cause the probability to exceed 1.

COMPLEMENTARY EVENTS

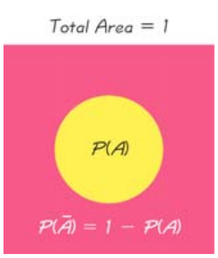
Note that events A and $\bar{A}$ are disjoint. Also, we can be absolutely certain that either A or $\bar{A}$ occurs. So we have

$$P(A \text{ or } \bar{A}) = P(A) + P(\bar{A}) = 1$$

RULE OF COMPLEMENTARY EVENTS

$$P(A) + P(\bar{A}) = 1$$
$$P(\bar{A}) = 1 - P(A)$$
$$P(A) = 1 - P(\bar{A})$$

VENN DIAGRAM FOR THE COMPLEMENT OF *A*



EXAMPLE

The data in the table below represent the income distribution of households in the US in 2016. *(Source: US Bureau of the Census)*

Annual Income	Number	Annual Income	Number
Less than $10,000	3,657	$50,000 to $74,999	14,547
$10,000 to $14,999	2,067	$75,000 to $99,999	11,631
$15,000 to $24,999	5,617	$100,000 to $149,999	14,271
$25,000 to $34,999	6,662	$150,000 to $199,999	6,959
$35,000 to $49,999	9,885	$200,000 or more	7,556
		Total	**82,854**

EXAMPLE (CONCLUDED)

1. Compute the probability that a randomly selected household earned $200,000 or more in 2016.
2. Compute the probability that a randomly selected household earned less than $200,000 in 2016.
3. Compute the probability that a randomly selected household earned at least $10,000 in 2016.

NOTATION

$$P(A \text{ and } B) = P(\text{event } A \text{ occurs in a first trial and event } B \text{ occurs in a second trial})$$

EXAMPLES

1. Suppose that you first toss a coin and then roll a die. What is the probability of obtaining a "Head" and then a "2"?
2. A bag contains 2 red and 6 blue marbles. Two marbles are randomly selected from the bag, one after the other, without replacement. What is the probability of obtaining a red marble first and then a blue marble?

CONDITIONAL PROBABILITY

- $P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred.
- Interpret $B|A$ as "event B occurring after event A has already occurred."

INDEPENDENT AND DEPENDENT EVENTS

- Two events A and B are **independent** if the occurrence of one event does not affect the probability of the occurrence of the other.
- Several events are **independent** if the occurrence of any does not affect the occurrence of the others.
- If A and B are not independent, they are said to be **dependent**.

INTUITIVE MULTIPLICATION RULE

To find the probability that event A occurs in one trial and B occurs in the another trial, multiply the probability of event A by the probability of event B, but be sure that the probability of event B is found by assuming event A has already occurred.

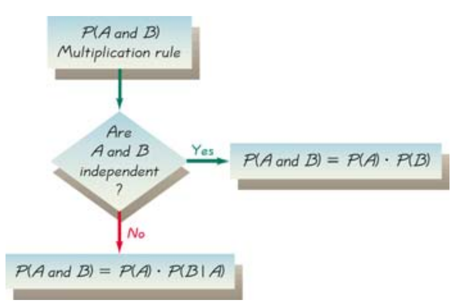
FORMAL MULTIPLICATION RULE

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

NOTE: If events A and B are ***independent***, then $P(B|A) = P(B)$ and the multiplication rule simplifies to

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

APPLYING THE MULTIPLICATION RULE



EXAMPLES

Determine whether the events A and B are independent or dependent and find $P(A \text{ and } B)$.

1. A: When a baby is born, it is a girl.
   B: When a second baby is born into a different family, it is also a girl.
2. A: When a day of the week is selected, it is a Saturday.
   B: When a *different* day of the week is selected, it is a Monday.
3. A: When the first digit (0 through 9) of a four-digit lottery number is chosen by someone buying a ticket, it is the same first digit that is later drawn in the official lottery.
   B: When the second digit of a four-digit lottery number is chosen by someone buying a ticket, it is the same digit that is later drawn in the official lottery.

EXAMPLES (CONTINUED)

4. A: When a survey subject is randomly selected from the 100 senators in the 116th congress, it is one of the 45 Democrats.
   B: When a second different senator is randomly selected, it is one of the 2 Independents.

ADDITIONAL EXAMPLES

1. What is the probability of drawing an “ace” from a standard deck of cards and then rolling a “7” on a pair of dice?
2. In the 116th Congress, the Senate consists of 24 women and 76 men, If a lobbyist for the tobacco industry randomly selected two different Senators, what is the probability that they were both men?
3. Repeat Example 2 except that three Senators are randomly selected.

EXCEPTION: TREATING DEPENDENT EVENTS AS INDEPENDENT

EXCEPTION: Some cumbersome calculations can be greatly simplified by using the common practice of treating events as independent when ***small samples*** are drawn from a ***large population***. In such cases, it is rare to select the same item twice.

5% GUIDELINE

Treating Dependent Events as Independent: 5% Guideline for Cumbersome Calculations

When calculations with sampling are very cumbersome and the sample size is not more than 5% of the size of the population, treat the selections as being ***independent*** (even if they are actually dependent).

EXAMPLE

In a survey of 10,000 African-Americans, it was determined that 27 had sickle cell anemia.

1. Suppose we randomly select one of the 10,000 African-Americans surveyed. What is the probability that he or she will have sickle cell anemia?
2. If two individuals from the group are randomly selected, what is the probability that both have sickle cell anemia?
3. Compute the probability of randomly selecting two individuals from the group who have sickle cell anemia, assuming independence.