Section 4-1

Basic Concepts of Probability

PROBABILITY

Probability is the measure of the likelihood that a given event will occur.

EVENTS AND SAMPLE SPACE

- An **event** is any collection of results or outcomes of a procedure.
- A **simple event** is an outcome or event that cannot be further broken down into simpler components.
- The **sample space** for a procedure consists of all possible ***simple*** events. That is, the sample space consists of all outcomes that cannot be broken down any further.

PROBABILITY

Probability is a measure of the likelihood that a given event will occur.

NOTATION:

- P denotes a probability.
- A, B, and C denote specific events.
- $P(A)$ denotes the "probability of event A occurring."

RULE 1: RELATIVE FREQUENCY APPROXIMATION OF PROBABILITY

Conduct (or observe) a procedure a large number of times, and count the number of times that event A actually occurs, $P(A)$ is then ***approximated*** as follows:

$$P(A) \approx \frac{\text{number of times } A \text{ occurred}}{\text{number of times procedure was repeated}}$$

This rule uses the **Law of Large Numbers**.

THE LAW OF LARGE NUMBERS

As a procedure is repeated again and again, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability.

CAUTION: The law of large numbers applies to behavior over a large number of trials, and it does not apply to one outcome. Don't make the foolish mistake of losing a large sum of money by incorrectly thinking that a string of losses increases the chances of a win on the next bet.

EXAMPLE

A fair die was tossed 563 times. The number "4" occurred 96 times. If you toss a fair die, what do you estimate the probability is for tossing a "4"?

RULE 2: CLASSICAL APPROACH TO PROBABILITY

Assume that a given procedure has n different simple events are ***equally likely***, and if event A can occur in s different ways, then

$$P(A) = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}} = \frac{s}{n}$$

CAUTION: When using the classical approach, always verify that the outcomes are ***equally likely***.

EXAMPLE

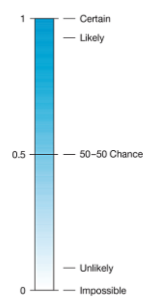
Find the probability of getting a "7" when a pair of dice is rolled.

RULE 3: SUBJECTIVE PROBABILITIES

$P(A)$, the probability of event A, is ***estimated*** by using knowledge of the relevant circumstances.

PROBABILITY LIMITS

- The probability of an impossible event is 0.
- The probability of an even that is certain to occur is 1.
- $0 \leq P(A) \leq 1$ for any event A



ROUNDING OFF PROBABILITIES

When expressing the value of a probability, either give the ***exact*** fraction or decimal or round off final decimal results to three significant digits.

Suggestion: When the probability is not a simple fraction such as 2/3 or 5/9, express it as a decimal so that the number can be better understood.

COMPLEMENTARY EVENTS

The complement of event A, denoted by $\bar{A}$, consists of all outcomes in which event A does ***not*** occur.

EXAMPLE

What is the probability of *not* rolling a "7" when a pair of dice is rolled?

RARE EVENT RULE FOR INFERENTIAL STATISTICS

If, under a given assumption, the probability of a particular observed event is extremely small and the observed event occurs *significantly less* than or *significantly greater* than what we typically expect with that assumption, we conclude that the assumption is probably not correct.

Statisticians use the **rare event rule for inferential statistics.**

USING PROBABILITIES WHEN RESULTS ARE SIGNIFICANTLY HIGH OR SIGNIFICATNLY LOW

- **Significantly high number of successes:** x successes in n trials is a *significantly high* number of successes if the probability of x or more success is unlikely with a probability of 0.05 or less. That is, x is a significantly high number of successes if $P(x \text{ or more}) \leq 0.05$.
- **Significantly low number of successes:** x successes in n trials is a *significantly low* number of successes if the probability of x or fewer success is unlikely with a probability of 0.05 or less. That is, x is a significantly low number of successes if $P(x \text{ or fewer}) \leq 0.05$.
- NOTE: The value of 0.05 is not absolutely rigid. Other values, such as 0.01, could be used to distinguish between results that could easily occur by chance and events that are very unlikely to occur by chance.

ODDS

- The **actual odds against** event A occurring are the ratio $P(\bar{A})/P(A)$, usually expressed in the form of a:b (or "a to b"), where a and b are integers having no common factors.
- The **actual odds in favor** of event A are ratio $P(A)/P(\bar{A})$, which is the reciprocal of the actual odds against that event. If the odds against A are a:b, then the odds in favor of A are b:a.
- The **payoff odds** against event A represent the ratio of the net profit (if you win) to the amount bet:

  payoff odds against A = (net profit) : (amount bet)

EXAMPLE

The American Statistical Association decided to invest some of its member revenue by buying a racehorse named Mean. Mean is entered in a race in which the actual probability of winning is 3/17.

(a) Find the actual odds against Mean winning.

(b) If the payoff odds are listed as 4:1, how much profit do you make if you bet $5 and Mean wins.