

Section 3-3

**Measures of Relative Standing and
Boxplots**

THE z SCORE

The **z score** (or **standard score** or **standardized value**) is the number of standard deviations that a given x value is above or below the mean. It is found using one of the following:

Sample: $z = \frac{x - \bar{x}}{s}$

Population: $z = \frac{x - \mu}{\sigma}$

Round z to two decimal places.

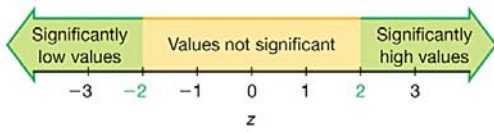
EXAMPLE

You are filling out an application for college. The application requires either your ACT or SAT score. You scored 26 on the ACT composite and 650 on the SAT. On the ACT exam, the composite mean score was 21 with a standard deviation of 5, while the SAT has a mean score of 514 with a standard deviation of 113. Which test score should you provide on the application? Why?

IMPORTANT PROPERTIES OF z SCORES

1. A z score is the number of standard deviations that a given value x is above or below the mean.
2. z scores are expressed as numbers with no units of measurement.
3. A data value is *significantly low* if its z score is less than or equal to -2 or the value is *significantly high* if its z score is greater than or equal to $+2$.
4. If an individual data value is less than the mean, its corresponding z score is a negative number

INTERPRETING z SCORES



Significant values are those with z scores ≤ -2 or ≥ 2

EXAMPLE

Adult males have heights with a mean of 69.4 inches and standard deviation of 2.8 inches. Actor Danny DeVito is 5 feet tall. Is he significantly short?

PERCENTILES

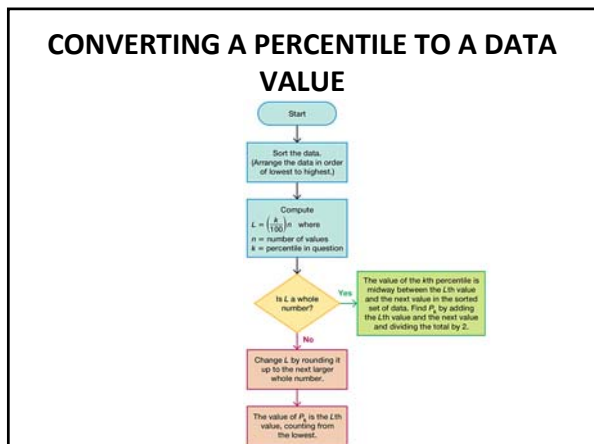
Percentiles are measures of location, denoted by $P_1, P_2, P_3, \dots, P_{99}$, which divide a set of data into 100 groups with about 1% of the values in each group.

FINDING THE PERCENTILE OF A DATA VALUE

Percentile of value $x = \frac{\text{number of values less than } x}{\text{total number of values}} \cdot 100$

NOTATION

- n = total number of values in the data set
- k = percentile being used
- L = locator that gives the *position* of a value
- P_k = k th percentile



- ### CONVERTING A PERCENTILE TO A DATA VALUE
1. Sort the data.
 2. Compute $L = \frac{k}{100} \cdot n$
 3. If L is a whole number, the k^{th} percentile is midway between the L^{th} value and the next value. It is computed by $\frac{x_L + x_{L+1}}{2}$
 4. If L is not a whole number, round up to the next whole number. The k^{th} percentile is the L^{th} value counting from the lowest.

QUARTILES

Quartiles are measures of location, denoted Q_1 , Q_2 , and Q_3 , which divide a set of data into four groups with about 25% of the values in each group.

The quartiles are given when “1-Var Stats” are run on the TI-83/84 calculators.

COMPUTING QUARTILES

- Note that the first quartile is the same as the 25th percentile. So to compute Q_1 , you compute P_{25} .
- Note that the second quartile (or median) is the same as the 50th percentile. So to compute Q_2 , you compute P_{50} .
- The third quartile is the same as the 75th percentile. So to compute Q_3 , you compute P_{75} .

5-NUMBER SUMMARY

For a set of data, the [5-number summary](#) consists of these five values:

1. Minimum value;
2. First quartile, Q_1 ;
3. Second quartile, Q_2 (same as the median);
4. Third quartile, Q_3 ; and
5. Maximum value.

EXAMPLE

Find the 5-number summary for Bank of Providence waiting times.

Bank of Providence (multiple waiting lines)	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0
--	-----	-----	-----	-----	-----	-----	-----	-----	-----	------

**BOXPLOTS
(BOX-AND-WHISKER DIAGRAMS)**

Boxplots are good for revealing:

1. center of the data
2. spread of the data
3. distribution of the data
4. presence of outliers

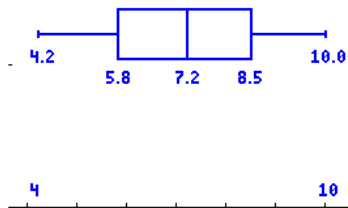
Boxplots are also excellent for comparing two or more data sets.

CONSTRUCTING A BOXPLOT

1. Find the 5-number summary.
2. Construct a scale with values that include the minimum and maximum data values.
3. Construct a box (rectangle) extending from Q_1 to Q_3 , and draw a line in the box at the value of Q_2 (median).
4. Draw lines extending outward from the box to the minimum and maximum data values.

AN EXAMPLE OF A BOXPLOT

Bank of Providence (multiple waiting lines)	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0
--	-----	-----	-----	-----	-----	-----	-----	-----	-----	------



DRAWING A BOXPLOT ON THE TI-83/84

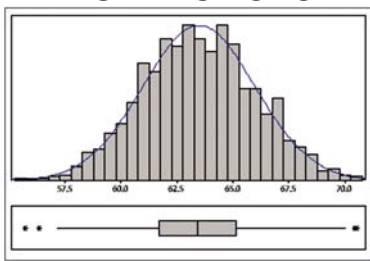
1. Press **STAT**; select **1:Edit...**
2. Enter your data values in **L1**. (Note: You could enter them in a different list.)
3. Press **2ND, Y=** (for **STATPLOT**). Select **1:Plot1**.
4. Turn the plot **ON**. For **Type**, select the boxplot (middle one on second row).
5. For **Xlist**, put L1 by pressing **2ND, 1**.
6. For **Freq**, enter the number 1.
7. Press **ZOOM**. Select **9:ZoomStat**.

EXAMPLE

Use boxplots to compare the waiting times at Jefferson Valley Bank and the Bank of Providence. Interpret your results.

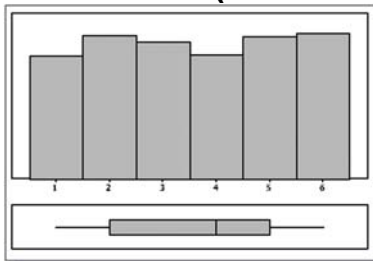
Jefferson Valley Bank (single waiting line)	6.5	6.6	6.7	6.8	7.1	7.3	7.4	7.7	7.7	7.7
Bank of Providence (multiple waiting lines)	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0

BOXPLOTS AND DISTRIBUTIONS



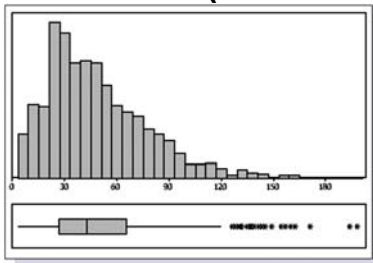
(a) Normal (bell-shaped) distribution
1000 heights (in.) of women

BOXPLOTS AND DISTRIBUTIONS (CONTINUED)



(b) Uniform distribution
1000 rolls of a die

BOXPLOTS AND DISTRIBUTIONS (CONCLUDED)



(c) Skewed distribution
Incomes (thousands of dollars) of 1000 statistics professors
