Section 6.7

Financial Models

COMPOUND INTEREST

The formula for **compound interest** (interest paid on both principal and interest) is an important application of exponential functions.

The amount A after t years due to a principal P invested at an annual percentage rate (APR) r compounded n times per year is

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

EXAMPLES

1. Suppose $1000 is deposited in an account with an annual percentage rate (APR) of 8%. How much money will be in the account after 7 years if interest is compounded
   (a) yearly,
   (b) quarterly,
   (c) monthly,
   (d) daily,
   (e) 1000 times a year,
   (f) 10,000 times a year?

2. Suppose $4000 is deposited in an account with an APR of 6% compounded monthly. How long will it take for there to be $15,000 in the account?

CONTINUOUS COMPOUNDING

As you noticed in Example 1 on the previous slide, when the number of compounding periods increases, the accumulated amount also increases but appears to approach some value. As the number of compounding periods approaches ∞, we say the interest is **compounded continuously**.

The amount A after t years due to a principal P invested at an annual percentage rate (APR) r compounded continuously is

$$A = P \cdot e^{rt}$$

EXAMPLES

3. Fred and Jane Sheffey have just invested $10,000 in a money market account with an APR of 7.65%. How much will they have in this account in 5 years if the interest is compounded continuously?

4. You put $5,000 in the bank at an APR of 12% compounded continuously.
   (a) Find a formula for the amount in the bank after t months.
   (b) Use your answer to part (a) to find the amount of money in the bank after 7 months.

EFFECTIVE RATE OF INTEREST

The **effective rate of interest** for an investment is the equivalent simple interest rate that would yield the same amount as compounding n times per year, or continuously, after ***one year***.

The effective rate of interest r_e of an investment earning an annual percentage rate (APR) r is given by

Compounding n time per year: $r_e = \left(1 + \frac{r}{n}\right)^n - 1$

Continuous Compounding: $r_e = e^r - 1$

EXAMPLES

5. Find the effective rate of interest of the following:
   (a) 10% compounded monthly
   (b) 3% compounded continuously
6. Which is the better investment?
   A: 4.7% compounded semiannually
   B: 4.65% compounded continuously

PRESENT VALUE

The **present value** of A dollars to be received at a future date is the principal that you would need to invest now so that it would grow to A dollars in the specified time period.

PRESENT VALUE (CONCLUDED)

The present values P of A dollars to be received after t years, assuming an annual percentage rate (APR) r compounded n times per year, is

$$P = A \cdot \left(1 + \frac{r}{n}\right)^{-nt}$$

If the interest is compounded continuously, then

$$P = A \cdot e^{-rt}$$

EXAMPLE

7. Find the principal needed to get \$1000 after 5 years if the interest is 8% compounded quarterly.