SECTION 6.6

Logarithmic and Exponential Equations

EXPONENTIAL EQUATION

An **exponential equation** is an equation in which the variable is located in the exponent.

For example: $3^{x-2} = 81$

EQUALITY-OF-EXPONENTS THEOREM

Theorem: If $a^u = a^v$, then $u = v$, provided that $a > 0$ and $a \neq 1$.

NOTE: This theorem can be used to solve *some* exponential equations. In particular, it can be used to solve equations in which both sides can be expressed as the same base raised to different powers.

Example: Solve $3^{x-2} = 81$

SOLVING EXPONENTIAL EQUATIONS

To solve an exponential equation, you can take the log or ln of both sides. This will allow the exponent to be moved in front of the logarithm.

EXAMPLES

1. Solve: $8^x = 700$
2. Solve: $3^{2x} = 56$
3. Solve: $5^{x+3} = 102$
4. Solve: $4^{3x+2} = 3^{2x+3}$

LOGARITHMIC EQUATIONS

A **logarithmic equation** is an equation in which the variable is located inside a logarithm.

For example: $\log(9x + 1) = 3$

SOLVING LOGARITHMIC EQUATIONS

Some logarithmic equations can be solved by converting the logarithmic form into the exponential form. Sometimes you may first need to use properties of logarithms to write one side as a single logarithm.

When solving logarithmic equations, you MUST ALWAYS check your solutions because some of them may not work.

EXAMPLES

1. Solve: $\log(9x+1)=3$
2. Solve: $\log_4 x + \log_4(x-30)=3$
3. Solve: $\log_3(x+4)=2+\log_3(2x-1)$

SOLVING LOGARITHMIC EQUATIONS THAT ARE NOT EQUAL TO A CONSTANT

If a logarithmic equation is not equal to (or cannot be made to be equal to) a constant, then use equality property of logarithms which says

$\log_a M = \log_a N$ if, and only if, $M=N$.

Example:

Solve: $\ln(2x+3)=\ln(4x+6)-\ln(1-x)$