Section 6.5

Properties of Logarithms

LOGARITHIMIC PROPERTIES

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $a^{\log_a M} = M$
4. $\log_a a^r = r$
5. $\log_a(MN) = \log_a M + \log_a N$
6. $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
7. $\log_a(M^r) = r \log_a M$
8. $\log_a\left(\frac{1}{N}\right) = -\log_a N$
9. $\log_a M = \log_a N \iff M = N$

EXAMPLES

Use the properties of logarithms to write each expression as a sum and/or difference of logarithms. Express powers as factors.

1. $\log_2(5x^3yz^4)$

2. $\log_6\left(\dfrac{2y^3}{x^4\sqrt{z}}\right)$

EXAMPLES

Use the properties of logarithms to rewrite each expression as a single logarithm.

1. $\frac{1}{2}\log_5 z - 3\log_5 y - 2\log_5(x+5)$

2. $3\ln(y-2) - 2\ln(y-3) + 4\ln y$

CHANGE OF BASE FORMULA

If M, a, and b are positive real numbers with $a \neq 1$ and $b \neq 1$, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

In particular, if M and a are positive real numbers and $a \neq 1$, then

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

EXAMPLES

Use the Change of Base formula and a calculator to evaluate each logarithm. Round your answer to the nearest thousandth.

1. $\log_4 150$

2. $\log_{16} 1000$