Section 6.4
Logarithmic Functions

LOGARITHMS
You will observe that exponential functions have the property that for a given $y$-value in the range there is only one $x$-value from which it comes. This means that if we know $y$ we can uniquely determine $x$. The process of find the $x$ given $y$ is called taking the logarithm of the number $y$.

Exponents and logarithms convey the same information but in different forms.

EXPONENTIAL AND LOGARITHMIC FORMS
• The exponential form of $y = \log_a x$ is $a^y = x$.
• The logarithmic form of $a^y = x$ is $y = \log_a x$.

LOGARITHMIC FUNCTIONS
The logarithmic function with base $a$, where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as "$y$ is the logarithm to the base $a$ of $x$") and is defined by

$$y = \log_a x \quad \text{if and only if} \quad x = a^y$$

The domain of the logarithmic function $y = \log_a x$ is $\{x \mid x > 0\}$ or in interval notation $(0, \infty)$. Observe that logarithmic and exponential functions have the roles of $x$ and $y$ reversed. This means their domains and ranges switch roles.

GRAPHING LOGARITHMIC FUNCTIONS
To quickly graph the logarithmic function $y = \log_a x$
plot points for $x = \frac{1}{a}, 1, a$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{a}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$a$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

DOMAIN AND RANGE OF LOGARITHMIC FUNCTION
• Domain of the logarithmic function = Range of the exponential function = $(0, \infty)$
• Range of the logarithmic function = Domain of the exponential function = $(-\infty, \infty)$

$y = \log_a x$ (defining equation: $x = a^y$)
Domain: $0 < x < \infty$ Range: $-\infty < x < \infty$
PROPERTIES OF $f(x) = \log_a x$

- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$
- The $x$-intercept of the graph is $(1,0)$. There is no $y$-intercept.
- Vertical Asymptote: $x = 0$
- The graph of $f$ contains the points $(1,-1)$, $(0,1)$, and $(a,1)$.
- Increasing if $a > 1$
- Decreasing if $0 < a < 1$
- The graph is smooth and continuous, with no corners or gaps.

DOMAIN OF A GENERAL LOGARITHMIC FUNCTIONS

Since the logarithm of a negative number and the logarithm of zero cannot be taken, the argument of a logarithmic function must always be positive. That is, if $Z$ is an algebraic expression in $x$, the domain of

$$f(x) = \log_a Z$$

is the set of numbers such that $Z > 0$.

COMMON AND NATURAL LOGARITHMS

Logarithms with a base of 10 are called common logarithms. We denote this by log $x$. That is,

$$\log x = \log_{10} x$$

Logarithms with a base of $e$ are called natural logarithms. We denote this by ln $x$. That is,

$$\ln x = \log_e x$$

LOGARITHMIC EQUATIONS

Equations that contain logarithms are called logarithmic equations. Some logarithmic equations can be solved by converting them to exponential form. However, when solving logarithmic equations, you must always check your solutions.