Section 6.4

Logarithmic Functions

LOGARITHIMS

You will observe that exponential functions have the property that for a given y-value in the range there is only one x-value from which it comes. This means that if we know y we can uniquely determine x. The process of find the x given y is called taking the **logarithm** of the number y.

Exponents and logarithms convey the same information but in different forms.

EXPONENTIAL AND LOGARITHMIC FORMS

- The exponential form of $y = \log_a x$ is $a^y = x$.
- The logarithmic form of $a^y = x$ is $y = \log_a x$.

LOGARITHMIC FUNCTIONS

The **logarithmic function with base a**, where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as "y is the logarithm to the base a of x") and is defined by

$$y = \log_a x \quad \text{if and only if} \quad x = a^y$$

The domain of the logarithmic function $y = \log_a x$ is $\{x \mid x > 0\}$ or in interval notation $(0, \infty)$.

Observe that logarithmic and exponential functions have the roles of x and y reversed. This means their domains and ranges switch roles.

GRAPHING LOGARITHMIC FUNCTIONS

To quickly graph the logarithmic function

$$y = \log_a x$$

plot points for $x = \frac{1}{a}$, 1, and a.

x	y
$\frac{1}{a}$	−1
1	0
a	1

DOMAIN AND RANGE OF LOGARITHMIC FUNCTION

- Domain of the logarithmic function = Range of the exponential function = $(0, \infty)$
- Range of the logarithmic function = Domain of the exponential function = $(-\infty, \infty)$

$y = \log_a x$ (defining equation: $x = a^y$)

Domain: $0 < x < \infty$ Range: $-\infty < x < \infty$

PROPERTIES OF $f(x) = \log_a x$

- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$
- The x-intercept of the graph is $(1, 0)$. There is no y-intercept.
- Vertical Asymptote: $x = 0$
- The graph of f contains the points $\left(\frac{1}{a}, -1\right)$, $(1,0)$, and $(a, 1)$.
- Increasing if $a > 1$
- Decreasing if $0 < a < 1$
- The graph is smooth and continuous, with no corners or gaps.

DOMAIN OF A GENERAL LOGARITHMIC FUNCTIONS

Since the logarithm of a negative number and the logarithm of zero cannot be taken, ***the argument of a logarithmic function must always be positive***. That is, if Z is an algebraic expression in x, the domain of

$$f(x) = \log_a Z$$

is the set of numbers such that $Z > 0$.

COMMON AND NATURAL LOGARITHMS

Logarithms with a base of 10 are called **common logarithms**. We denote this by $\log x$. That is,

$$\log x = \log_{10} x$$

Logarithms with a base of e are called **natural logarithms**. We denote this by $\ln x$. That is,

$$\ln x = \log_e x$$

LOGARITHMIC EQUATIONS

Equations that contain logarithms are called **logarithmic equation**. Some logarithmic equations can be solved by converting them to exponential form. However, when solving logarithmic equations, ***you must always check your solutions***.