Sections 6.3

Exponential Functions

LAWS OF EXPONENTS

If $s, t, a,$ and b are real numbers with a positive and b positive, then

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s$$

$$\left(\frac{a}{b}\right)^s = \frac{a^s}{b^s} \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s$$

$$1^s = 1 \qquad a^0 = 1$$

EXPONENTIAL FUNCTION

The **exponential function** is a function of the form

$$f(x) = C \cdot a^x$$

where a is a positive number ($a > 0$), $a \neq 1$, and $C \neq 0$ is a real number. The domain of f is the set of real numbers. The base a is called the **growth factor**, and because, $f(0) = C \cdot a^0 = C$, the number C is called the **initial value**.

NOTE: Do not confuse exponential and power functions

- $f(x) = x^2$ (*power function*)
- $f(x) = 2^x$ (*exponential function*)

GRAPHING EXPONENTIAL FUNCTIONS

To graph the exponential function

$$f(x) = a^x$$

plot points for $x = -1, 0,$ and 1.

x	$f(x)$
-1	$\frac{1}{a}$
0	1
1	a

A THEOREM

Theorem: For an exponential function

$$f(x) = C \cdot a^x, \; a > 0, \; a \neq 1, C \neq 0$$

if x is any real number, then

$$\frac{f(x+1)}{f(x)} = a \quad \text{or} \quad f(x+1) = a \cdot f(x)$$

PROPERTIES OF $f(x) = a^x$

- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- There are no x-intercepts; the y-intercept is $(0, 1)$.
- Horizontal Asymptote: $y = 0$ (the x-axis)
- The graph contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
- Increasing if $a > 1$
- Decreasing if $0 < a < 1$
- The graph of f is smooth and continuous, with no corners or gaps.

GROWTH AND DECAY FUNCTIONS

If an exponential function is increasing, it is called an **exponential growth function**.

If an exponential function is decreasing, it is called an **exponential decay function**.

THE NUMBER *e*

The number e is called the **natural number** and is often the base of an exponential function. It is defined as the number that the expression

$$\left(1+\frac{1}{n}\right)^n$$

approaches as $n \to \infty$.

The decimal approximation of e is

$$e \approx 2.718281828459045$$

$$e \approx 2.7\ \ 1828\ \ 1828\ \ 45\ \ 90\ \ 45$$

THE NATURAL EXPONENTIAL FUNCTION

The exponential function defined by

$$f(x) = e^x$$

is called the **natural exponential function**.

EXPONENTIAL EQUATIONS

Equations that involve terms of the form a^x are often referred to as **exponential equations**. Such equations can sometimes be solved by using the following theorem.

Theorem: If $a^u = a^v$, then $u = v$.