Section 5.6

The Real Zeros of a Polynomial Function

THE DIVISION ALGORITHM FOR POLYNOMIALS

If $f(x)$ and $g(x)$ denote polynomial functions and if $g(x)$ is not the zero polynomial, there are unique polynomial functions $q(x)$ and $r(x)$ such that

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad f(x) = q(x) \cdot g(x) + r(x)$$

where $r(x)$ is either the zero polynomial or polynomial of degree less than that of $g(x)$.

DIVIDEND, DIVISOR, QUOTIENT, AND REMAINDER

In the equation on the previous slide,

- $f(x)$ is the **dividend**
- $g(x)$ is the **divisor**
- $q(x)$ is the **quotient**
- $r(x)$ is the **remainder**

THE REMAINDER THEOREM

Let $f(x)$ be a polynomial function. If $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

THE FACTOR THEOREM

Let f be a polynomial function. Then $x - c$ is a factor of $f(x)$ if and only if $f(c) = 0$.

The Factor Theorem consists of two separate statements.

1. If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.
2. If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.

THE NUMBER OF REAL ZEROS

A polynomial function cannot have more zeros than its degree.