Section 5.5

Polynomial and Rational Inequalities

SOLVING A POLYNOMIAL OR RATIONAL INEQUALITY

Step 1: Write the inequality so that the polynomial or rational expression \( f \) is on the left side and zero is on the right side in one of the following forms:

\[
\begin{align*}
  f(x) &> 0 & f(x) &\geq 0 \\
  f(x) &< 0 & f(x) &\leq 0
\end{align*}
\]

For rational expressions, be sure that the left side is written as a single quotient and find its domain.

SOLVING (CONTINUED)

Step 2: Determine the real numbers at which the expression \( f \) on the left side is equal to zero, and, if the expression is rational, the real numbers at which the expression \( f \) on the left side is undefined.

Step 3: Use the numbers from Step 2 to separate the real number into intervals.

SOLVING (CONCLUDED)

Step 4: Select a number in each interval and evaluate \( f \) at that number.

(a) If the value of \( f \) is positive, then \( f(x) > 0 \) for all numbers \( x \) in the interval.

(b) If the value of \( f \) is negative, then \( f(x) < 0 \) for all numbers \( x \) in the interval.

If the inequality is not strict, include the solutions of \( f(x) = 0 \) in the solution set. Be careful to exclude values of \( x \) where \( f \) is undefined.

TEST VALUES

The numbers selected in Step 4 are called test values because they are used to test whether the function is positive or negative in the interval.

The preceding method is sometimes called the test-value method for solving inequalities.