Section 4.3

Quadratic Functions and Their Properties

QUADRATIC FUNCTIONS

A **quadratic function** of x is a function that can be represented by an equation of the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$. The domain of a quadratic function is all real numbers.

This is called the **standard form** of a quadratic function.

GRAPHS OF QUADRATIC FUNCTIONS

- The graph of a quadratic function is a **parabola**.
- The parabola opens up if the coefficient of x^2 is positive.
- The parabola opens down if the coefficient of x^2 is negative.
- The **vertex of a parabola** is the lowest point on a parabola that opens up or the highest point on a parabola that opens down.
- The **axis of symmetry** is the vertical line passing through the vertex of a parabola.

VERTEX FORM OF QUADRATIC FUNCTIONS

Every quadratic function given by $f(x) = ax^2 + bx + c$ can be written in the **vertex form** of a quadratic function:

$$f(x) = a(x - h)^2 + k, \qquad a \neq 0$$

The graph of f is a parabola with vertex (h, k). The parabola opens up if a is positive, and it opens down if a is negative.

To find the vertex form of a quadratic function, use the **technique of completing the square**.

VERTEX FORMULA

The vertex of the graph of $f(x) = ax^2 + bx + c$ is

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

SUMMARY OF PROPERTIES OF THE GRAPH OF A QUADRATIC FUNCTION

$$f(x) = ax^2 + bx + c, \qquad a \neq 0$$

- Vertex $= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
- Axis of Symmetry: the line $x = -\frac{b}{2a}$
- Parabola opens up if is $a > 0$; the vertex is a minimum point.
- Parabola opens down if is $a < 0$; the vertex is a maximum point.

x-INTERCEPTS OF A QUADRATIC FUNCTION

1. If the discriminant $b^2 - 4ac > 0$, then graph of $f(x) = ax^2 + bx + c$ has two distinct x-intercepts so it crosses the x-axis in two places.
2. If the discriminant $b^2 - 4ac = 0$, then graph of $f(x) = ax^2 + bx + c$ has one x-intercept so it touches the x-axis in at its vertex.
3. If the discriminant $b^2 - 4ac < 0$, then graph of $f(x) = ax^2 + bx + c$ has no x-intercept so it does not cross or touch the x-axis.

MAXIMUM OR MINIMUM VALUE OF A QUADRATIC FUNCTION

- If a is positive, then the vertex (h, k) is the lowest point on the graph of $f(x) = a(x - h)^2 + k$, and the y-coordinate k of the vertex is the **minimum value** of the function f.
- If a is negative, then the vertex (h, k) is the highest point on the graph of $f(x) = a(x - h)^2 + k$, and the y-coordinate k of the vertex is the **maximum value** of the function f.
- In either case, the maximum or minimum value is achieved when $x = h$.