Section 3.1

Functions

RELATIONS

A **relation** is *any* correspondence between two sets.

If x and y are two elements in these sets and if a relationship exists between x and y, then we say that x **corresponds** to y.

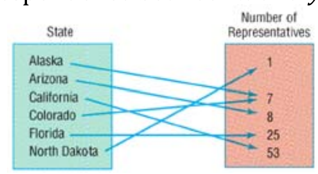
WAYS TO REPRESENT A RELATION

A relation can be represented in four ways:

- A listing of the ordered pairs, many times as a table
- A graph
- An equation
- A mapping

MAPPINGS

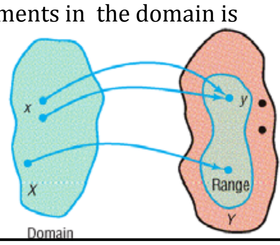
A **map** illustrates a relation by using two sets and drawing arrows between them to show the correspondence between x and y.



{(Alaska, 7), (Arizona, 8), (California, 53), (Colorado, 7), (Florida, 25), (North Dakota, 1)}

FUNCTIONS

Let X and Y be two nonempty sets. A **function** from X into Y is a relation that associates with each element of X exactly one element of Y. The set X is called the **domain** of the function. The set of all images of the elements in the domain is called the **range**.



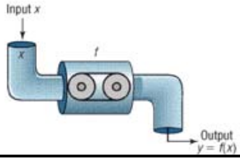
DETERMINING IF A RELATION IS A FUNCTION

To determine if a relation is a function check to see if the same x-value is always paired with the same y-value.

FUNCTION NOTATION

To emphasize that y is a function of x, the notation $y = f(x)$ is used. This does ***NOT*** mean f and x are multiplied. It does mean that the function f with input x produces the output y.

$$f(\text{input}) = \text{output}$$



INDEPENDENT AND DEPENDENT VARIABLES

For the function, $y = f(x)$,

- the variable x is called the **independent variable**, because it can be assigned any permissible number from the domain.
- the variable y is called the **dependent variable**, because its value depends on x.

NOTE: The independent variable is also called the **argument** of the function.

THE DIFFERENCE QUOTIENT

The expression

$$\frac{f(x+h) - f(x)}{h}, \qquad h \neq 0$$

is called the **difference quotient**. It is used in Calculus to study how a function value changes as the independent variable changes.

DETERMINING IF AN EQUATION IS A FUNCTION

To determine if an equation represents a function, solve the equation for y to see if there are two y-values for one x-value

DOMAIN AND RANGE

The **domain** can be thought of as the set of *acceptable* inputs of a function.

The **range** is the set of outputs.

DETERMINING DOMAIN FROM EQUATIONS

To find the domain given an equation, look for acceptable inputs.

Common Examples:

1. Fractions: all numbers except those that make the denominator zero.
2. Even Roots: all numbers that make the radicand zero or positive.

OPERATIONS ON FUNCTIONS

Sum: $(f+g)(x) = f(x) + g(x)$

Difference: $(f-g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x)g(x)$

Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \; g(x) \neq 0$

DOMAINS

- The domain for $f+g$, $f-g$, and fg is the intersection of the domains of f and g; that is, $D_f \cap D_g$.
- The domain for f/g is the intersection of the domains of f and g, except for those numbers x such that $g(x) = 0$; that is, $D_f \cap D_g - \{x \mid g(x) = 0\}$.