Section 2.4

Circles

**DEFINITION OF A CIRCLE**

A circle is a set of points in the \(xy\)-plane that are a fixed distance \(r\) from a fixed point \((h, k)\). The fixed distance \(r\) is called the radius, and the fixed point \((h, k)\) is called the center of the circle.

**STANDARD FORM OF AN EQUATION OF A CIRCLE**

The standard form of an equation of a circle with radius \(r\) and center \((h, k)\) is

\[(x - h)^2 + (y - k)^2 = r^2\]

**CIRCLES WITH CENTER AT THE ORIGIN**

*Theorem:* The standard form of an equation of a circle of radius \(r\) with center at the origin \((0,0)\) is

\[x^2 + y^2 = r^2\]

*Definition:* If the radius \(r = 1\), the circle whose center is at the origin is called the unit circle and has equation

\[x^2 + y^2 = 1\]

**GENERAL FORM OF THE EQUATION OF A CIRCLE**

When its graph is a circle, the equation

\[x^2 + y^2 + ax + by + c = 0\]

is referred to as the general form of the equation of a circle.

**WRITING AN EQUATION OF A CIRCLE IN STANDARD FORM**

To write the equation of a circle in standard form:

1. Collect all variable terms on the left side of the equation and all constant terms on the right side.
2. Group the “x”-terms together and the “y”-terms together.
3. Complete the square for both \(x\)- and \(y\)-terms.
4. Write \(x\)- and \(y\)-terms as the perfect squares of binomials.