

Section 2.3

Lines

SLOPE OF A LINE

Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope m** of the nonvertical line L containing P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

If $x_1 = x_2$, L is a **vertical line** and the slope m of L is **undefined** (since this results in division by 0).

OTHER WAYS TO REMEMBER THE SLOPE FORMULA

The slope formula is some times remembered as

$$m = \frac{\text{rise}}{\text{run}}$$

or

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

SLOPE AND THE SHAPE OF A LINE

1. When the slope of a line is positive ($m > 0$), the line slants upward from left to right.
2. When the slope of a line is negative ($m < 0$), the line slants downward from left to right.
3. When the slope of a line is zero ($m = 0$), the line is horizontal.
4. When the slope is undefined, the line is vertical.

EQUATION OF A VERTICAL LINE

A vertical line is given by an equation of the form

$$x = a$$

where a is the x -intercept.

POINT-SLOPE EQUATION OF A LINE

An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

EQUATION OF A HORIZONTAL LINE

A horizontal line is given by an equation of the form

$$y = b$$

where b is the y -intercept.

SLOPE-INTERCEPT FORM OF AN EQUATION FOR A LINE

An equation of a line with slope m and y -intercept b is

$$y = mx + b$$

GENERAL FORM OF AN EQUATION OF A LINE

The equation of a line is in [general form](#) (or [standard form](#)) when it is written as

$$Ax + By = C$$

where A , B , and C are real numbers and A and B are not both 0.

PARALLEL LINES

Criterion for Parallel Lines: Two nonvertical lines are parallel if and only if their slopes are equal and they have different y -intercepts.

- If two nonvertical lines are parallel, then their slopes are equal and they have different y -intercepts.
- If two nonvertical lines have equal slopes and different y -intercepts, then they are parallel.

PERPENDICULAR LINES

Criterion for Perpendicular Lines: Two nonvertical lines are perpendicular if and only if the product of their slopes is -1 .

- If two nonvertical lines are perpendicular, then the product of their slopes is -1 .
- If two nonvertical lines have slopes whose product is -1 , then they are perpendicular.