Section 2.1

The Distance and Midpoint Formulas

THE CARTESIAN COORDINATE SYSTEM

Quadrant II
$(-,+)$

Quadrant I
$(+,+)$

origin
$(0,0)$

Quadrant III
$(-,-)$

Quadrant IV
$(+,-)$

y

x
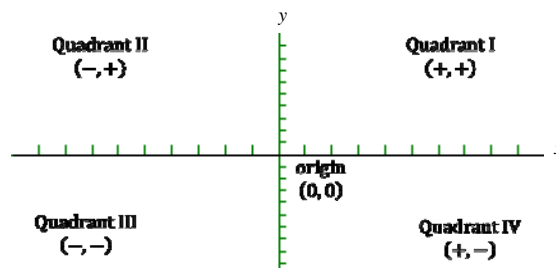
THE DISTANCE FORMULA

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

To compute the distance between two points, find the difference in the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.

EXAMPLE

Consider the points $A = (-2, 5)$, $B = (12, 3)$, and $C = (10, -11)$.

(a) Plot each point and form the triangle ABC.

(b) Find the length of each side of triangle ABC.

(c) Verify that triangle ABC is a right triangle.

(d) Compute the area of triangle ABC.

THE MIDPOINT FORMULA

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is given by

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

To find the midpoint of a line segment, average the x-coordinates of the endpoints, and average the y-coordinates of the endpoints.