Section 1.5

Solving Inequalities

INTERVALS

- A **closed interval**, denoted by $[a, b]$, consists of all real numbers x for which $a \le x \le b$.
- An **open interval**, denoted by (a, b), consists of all real numbers x for which $a < x < b$.
- The **half-open**, or **half-closed**, **intervals** are $(a, b]$, consisted of all real numbers x for which $a < x \le b$, and $[a, b)$, consisting of all real numbers x for which $a \le x < b$.
- In each of these definitions, the number a is called the **left endpoint**, and the number b is called the **right endpoint** of the interval.

NONNEGATIVE PROPERTY OF INEQUALITIES

For any real number a,

$$a^2 \ge 0$$

ADDITION PROPERTY OF INEQUALITIES

For real numbers a, b, and c,

If $a < b$, then $a + c < b + c$.

If $a > b$, then $a + c > b + c$.

This property says that the direction of the inequality remains unchanged if the same number is added to each side. The same is true for subtracting a number since subtracting is equivalent to adding a negative number.

MULTIPLICATION PROPERTY FOR INEQUALITIES

For real numbers a, b, and c,

If $a < b$ and $c > 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

If $a > b$ and $c > 0$, then $ac > bc$.
If $a > b$ and $c < 0$, then $ac < bc$.

This property says that if the inequality is multiplied by a positive number the inequality sign stays the same. If it is multiplied by a negative number, the direction of the inequality sign changes.

RECIPROCAL PROPERTY FOR INEQUALITIES

If $a > 0$, then $\frac{1}{a} > 0$. If $\frac{1}{a} > 0$, then $a > 0$.

If $a < 0$, then $\frac{1}{a} < 0$. If $\frac{1}{a} < 0$, then $a < 0$.

This property says that the reciprocal of a positive real number is positive and the reciprocal of a negative real number is negative.

PROCEDURES THAT LEAVE INEQUALITY SYMBOL UNCHANGED

1. Simplify both sides of the inequality by combining like terms and eliminating parentheses.
2. Add or subtract the same expression on both sides of the inequality.
3. Multiply or divide both sides of the inequality by the same ***positive*** expression.

PROCEDURES THAT REVERSE THE SENSE OR DIRECTION OF THE INEQUALITY SYMBOL

1. Interchange the two sides of the inequality.
2. Multiply or divide both sides of the inequality by the same ***negative*** expression.
3. If you take the reciprocal of both sides of an inequality.