

## Section 1.2

### Quadratic Equations

## QUADRATIC EQUATIONS

- A [quadratic equation](#) is an equation equivalent to one of the form

$$ax^2 + bx + c = 0$$

where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ .

- The form above is called the [standard form](#) of a quadratic equation.
- A quadratic equation is also called a [second-degree equation](#).

## SOLVING QUADRATIC EQUATIONS

Quadratic equations can be solved by several methods.

- By factoring and using the Zero-Product Property
- By taking square roots
- By completing the square
- By using the Quadratic Formula

## SOLVING A QUADRATIC EQUATION BY FACTORING

- Write the quadratic equation in standard form.
- Factor the quadratic expression.
- Use the Zero-Product Property to set each factor equal to 0.
- Solve the linear equations resulting from Step 3.
- Check your solution(s).

## REPEATED SOLUTIONS

When the left side factors into two linear equations with the same solution, the quadratic equation is said to have a [repeated solution](#). We also call this solution a [root of multiplicity 2](#), or a [double root](#).

## SOLVING BY THE SQUARE ROOT METHOD

- Write the quadratic in the form of  $A^2 = p$ , where  $A$  is a linear expression and  $p$  is a real number.
- Take the square root of both sides.
- Solve the resulting linear equations if necessary.
- Check your solution(s).

## SQUARES OF A BINOMIAL

Square of a Binomial	Perfect-Square Trinomial
$(x + 5)^2$	$x^2 + 10x + 25$
$(x - 4)^2$	$x^2 - 8x + 16$

In each perfect-square trinomial, the coefficient of  $x^2$  is 1, and the constant term of the perfect-square trinomial is the square of half the coefficient of its  $x$  term.

## COMPLETING THE SQUARE

Adding to a binomial of the form  $x^2 + bx$ , the constant that makes the binomial a perfect square trinomial is called completing the square.

To complete the square,

1. Divide the coefficient of  $x$  by 2; that is,  $\frac{b}{2}$ .
2. Square the result of Step 1.
3. Add the result of Step 2.

The resulting trinomial is a perfect square.

## SOLVING A QUADRATIC EQUATION BY COMPLETING THE SQUARE

1. Write the quadratic equation in the form  $ax^2 + bx = c$ .
2. Factor the coefficient of  $x^2$  out of the left side of the equation.
3. Complete the square of the resulting binomial. Be sure to maintain equality by adding the same quantity to both sides.
4. Solve the equation resulting from Step 3 by the Square Root Method.
5. Check your solution(s).

## SOLVING A QUADRATIC EQUATION BY THE QUADRATIC FORMULA

Consider the equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

If  $b^2 - 4ac < 0$ , this equation has no real solution.

If  $b^2 - 4ac \geq 0$ , the real solution(s) of this equation is (are) given by the [quadratic formula](#).

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## THE DISCRIMINANT

In the Quadratic Formula, the quantity  $b^2 - 4ac$  is called the [discriminant](#).

This discriminant can tell us if the equation has real solutions and how many to expect.

## USING THE DISCRIMINANT

For a quadratic equation  $ax^2 + bx + c = 0$ :

1. If  $b^2 - 4ac > 0$ , there are two unequal real solutions.
2. If  $b^2 - 4ac = 0$ , there is a repeated real solution, a root of multiplicity 2.
3. If  $b^2 - 4ac < 0$ , there is no real solution.