The Method of Disks/Washers

<u>NOTE</u>: In the Method of Disks/Washers the slices are always ***<u>perpendicular</u>*** to the axis of rotation.

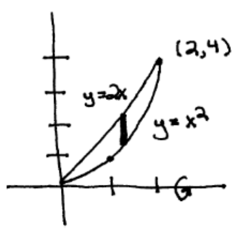
<u>Axis of Rotation Horizontal</u>

$y = c$

Slices are vertical

Integrate with respect to x (all equations in terms of x)

<u>Example</u>: Find the volume when the first-quadrant portion of the region bounded by $y = x^2, y = 2x$ is rotated about the x-axis.



$$V = \pi \int_0^2 [(2x)^2 - (x^2)^2] dx$$
$$= \pi \int_0^2 (4x^2 - x^4) dx$$
$$\vdots$$
$$= \frac{64\pi}{15}$$

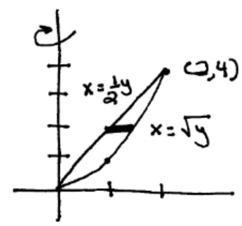
<u>Axis of Rotation Vertical</u>

$x = d$

Slices are horizontal

Integrate with respect to y (all equations in terms of y)

<u>Example</u>: Find the volume when the first-quadrant portion of the region bounded by $y = x^2, y = 2x$ is rotated about the y-axis.



$$V = \pi \int_0^4 \left[\left(\sqrt{y}\right)^2 - \left(\tfrac{1}{2}y\right)^2 \right] dy$$
$$= \pi \int_0^4 \left(y - \tfrac{1}{4}y^2\right) dy$$
$$\vdots$$
$$= \frac{8\pi}{3}$$

THE METHOD OF SHELLS

NOTE: In the Method of Shells the slices are always ***parallel*** to the axis of rotation.

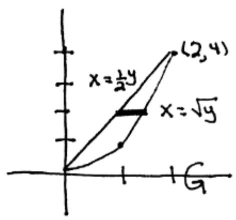
Axis of Rotation Horizontal

$y = c$

Slices are horizontal

Integrate with respect to y (all equations in terms of y)

Example: Find the volume when the first-quadrant portion of the region bounded by $y = x^2, y = 2x$ is rotated about the x-axis.



$$V = 2\pi \int_0^4 y\left(\sqrt{y} - \tfrac{1}{2}y\right) dy$$

$$= 2\pi \int_0^4 \left(y^{3/2} - \tfrac{1}{2}y^2\right) dy$$

$$\vdots$$

$$= \frac{64\pi}{15}$$

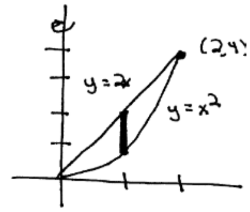
Axis of Rotation Vertical

$x = d$

Slices are vertical

Integrate with respect to x (all equations in terms of x)

Example: Find the volume when the first-quadrant portion of the region bounded by $y = x^2, y = 2x$ is rotated about the y-axis.



$$V = 2\pi \int_0^2 x(2x - x^2) dx$$

$$= 2\pi \int_0^2 (2x^2 - x^3) dx$$

$$\vdots$$

$$= \frac{8\pi}{3}$$