

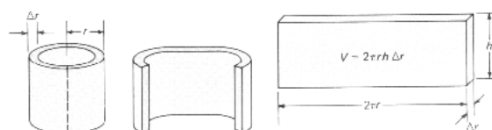
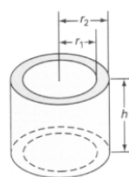
## Section 5.3

### Volumes by Cylindrical Shells

### THE VOLUME OF CYLINDRICAL SHELL

$$V = 2\pi r h \Delta r$$

where  $r$  is the average radius,  $h$  is the height, and  $\Delta r$  is the thickness.



### VOLUME BY CYLINDRICAL SHELLS

1. If the solid consists of concentric **vertical** shells between  $x = a$  and  $x = b$ , we find the radius  $r(x)$  and height  $h(x)$  of the shell at  $x$ , and the volume is

$$V = 2\pi \int_a^b r(x)h(x)dx$$

2. If the solid consists of concentric **horizontal** shells between  $y = c$  and  $y = d$ , we find the radius  $r(y)$  and length  $l(y)$  of the shell at  $y$ , and the volume is

$$V = 2\pi \int_c^d r(y)l(y)dy$$

### PROCEDURE FOR THE METHOD OF SHELLS

1. Sketch the region. Label intersection points, if any.
2. Draw a slice and label the radius, height, and thickness.
3. Sum the volume of all shells; that is, set up and evaluate a definite integral.