Section 5.2

Volumes

DEFINITION OF VOLUME USING VERTICAL SLICES

Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the volume of S is

$$V = \int_a^b A(x)dx$$

DEFINITION OF VOLUME USING HORIZONTAL SLICES

Let S be a solid that lies between $y = c$ and $y = d$. If the cross-sectional area of S in the plane, through y and perpendicular to the y-axis, is $A(y)$, where A is a continuous function, then the volume of S is

$$V = \int_c^d A(y)dy$$

SOME USEFUL AREAS

1. If the cross-section is a ***disk***, we find the radius of the disk (in terms of x or y) and use

$$A = \pi(\text{radius})^2$$

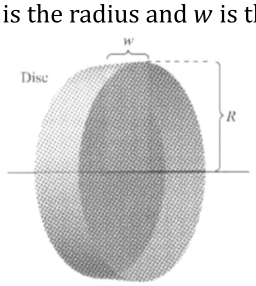
2. If the cross-section is a ***washer***, we find the ***inner*** radius and the ***outer*** radius of the washer (in terms of x or y) and use

$$A = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$$
$$= \pi[(\text{outer radius})^2 - (\text{inner radius})^2]$$

VOLUME OF A DISK

$$V = \pi R^2 w$$

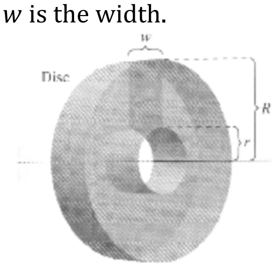
where R is the radius and w is the width



VOLUME OF A WASHER

$$V = \pi(R^2 - r^2)w$$

where R is the outside radius, r is the inside radius, and w is the width.



VOLUME BY DISKS

1. If the solid consists of adjacent ***vertical*** disks between $x = a$ and $x = b$, we find the radius $R(x)$ of the disk at x, and the volume is

$$V = \pi \int_a^b [R(x)]^2 dx$$

2. If the solid consists of adjacent ***horizontal*** disks between $y = c$ and $y = d$, we find the radius $R(y)$ of the disk at y, and the volume is

$$V = \pi \int_c^d [R(y)]^2 dy$$

VOLUME BY WASHERS

1. If the solid consists of adjacent ***vertical*** washers between $x = a$ and $x = b$, we find the outside radius $R(x)$ and inside radius $r(x)$ of the washer at x, and the volume is

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$$

2. If the solid consists of adjacent ***horizontal*** washers between $y = c$ and $y = d$, we find the outside radius $R(y)$ and inside radius $r(y)$ of the disk at y, and the volume is

$$V = \pi \int_c^d ([R(y)]^2 - [r(y)]^2) dy$$

PROCEDURE FOR FINDING VOLUMES BY THE METHOD OF DISKS

1. Sketch the region. Label intersection points.
2. Draw a slice and label the outside radius, inside radius (if necessary), and width.
3. Sum the volume of all disks (or washers); that is, set up and evaluate a definite integral.