Section 5.1

Area Between Curves

AREA BETWEEN CURVES USING VERTICAL SLICES, PART 1

The area A of the region bounded by the curves $y = f(x),\ y = g(x),$ and the lines $x = a,\ x = b,$ where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$ is

$$A = \int_a^b [f(x) - g(x)]dx$$

PROCEDURE FOR FINDING THE AREA

1. Sketch the region. Label intersection points, if any.
2. Slice it into thin pieces (rectangles); label a typical piece.
3. Sum the areas of all the rectangles; that is, set up and evaluate a definite integral.

AREA BETWEEN CURVES USING VERTICAL SLICES, PART 2

The area between the curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is

$$A = \int_a^b |f(x) - g(x)|dx$$

COMMENTS:

1. The function f does not have the be "above" g.
2. To evaluate the above integral, we must split it into more than one integral, depending on which function is "on top."

AREA BETWEEN CURVES USING HORIZONTAL SLICES

1. The area A of the region bounded by the curves $x = f(y)$ and $x = g(y)$, and the lines $y = c$ and $y = d$, where f and g are continuous and $f(y) \geq g(y)$ for all y in $[c, d]$, is

$$A = \int_c^d [f(y) - g(y)]dy$$

2. The area A between the curves $x = f(y)$ and $x = g(y),$ and $y = c$ and $y = d$, is

$$A = \int_c^d |f(y) - g(y)|dy$$