Section 4.5

The Substitution Rule

FINDING ANTIDERIVATIVES

Because of the Fundamental Theorem, it is important to be able to find antiderivatives. However, most antiderivatives are not easy to find. For example,

$$\int 4x^3\sqrt{1+x^4}\,dx$$

To do this, we need something extra. We change from the old variable to a new variable through the use of differentials.

SUBSTITUTION

The process performed on the last problem is called ***u*-substitution** or simply **substitution**.

Substitution will work anytime we have a composite function with the derivative of the "inside" function multiplied; that is, when we have the form

$$\int f\big(g(x)\big)g'(x)dx$$

If $F'(x) = f(x)$, then

$$\int F'\big(g(x)\big)g'(x)dx = F\big(g(x)\big) + C$$

THE SUBSTITUTION RULE

Theorem: If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f\big(g(x)\big)g'(x)dx = \int f(u)du$$

The Substitution Rule says: **It is permissible to operate with dx and du after the integral as if they were differentials.**

THE SUBSTITUTION RULE FOR DEFINITE INTEGRALS

Theorem: If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f\big(g(x)\big)g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

INTEGRALS OF SYMMETRIC FUNCTIONS

<u>Theorem</u>: Suppose f is continuous on $[-a, a]$.

(a) If f is even (symmetric about the y-axis), then

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

(b) If f is odd (symmetric about the origin), then

$$\int_{-a}^{a} f(x)dx = 0$$