Section 4.4

Indefinite Integrals and the Net Change Theorem

INDEFINITE INTEGRALS

Recall from Section 3.9 that we used the notation $A_x f(x)$ to mean find the antiderivative of $f(x)$. However, this is ***not*** the most common notation for taking the antiderivative. The notation commonly used is

$$\int f(x)dx$$

Another name for the antiderivative is the **indefinite integral**.

A REMARK ABOUT THE DEFINITE INTEGRAL

$$\int f(x)dx = F(x) \quad \text{means} \quad F'(x) = f(x)$$

You should distinguish carefully between the *definite integral* and the *indefinite integral*.

A definite integral $\int_a^b f(x)dx$ is a number representing an area.

An indefinite integral $\int f(x)dx$ is a function (or family of functions) that is the antiderivative of the integrand.

INDEFINITE INTEGRAL FORMULAS

$$\int cf(x)dx = c\int f(x)dx \qquad \int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int kdx = kx + C \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C = \frac{1}{n+1}x^{n+1} + C \ (n \neq -1)$$

$$\int \sin x\, dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C$$

$$\int \sec^2 x\, dx = \tan x + C \qquad \int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C \qquad \int \csc x \cot x\, dx = -\csc x + C$$

THE NET CHANGE THEOREM

Theorem: The integral of a rate of change is the net change; that is,

$$\int_a^b F'(x)dx = F(b) - F(a)$$