# Section 4.3

**The Fundamental Theorem of Calculus**

## A FUNCTION DEFINED BY A DEFINITE INTEGRAL

The first part of the Fundamental Theorem of Calculus deals with functions defined by a definite integral.

Let $f(t)$ be a continuous function defined on the interval $[a, b]$. If $x$ varies between $a$ and $b$, we can define the function $g$ by

$$g(x) = \int_a^x f(t)dt$$

The function $g$ is sometimes called an **accumulation function** since gives the area "accumulated so far."

## THE FUNDAMENTAL THEOREM OF CALCULUS, PART 1 (FTC1)

**Theorem**: If $f$ is continuous on $[a, b]$, then the function $g$ defined by

$$g(x) = \int_a^x f(t)dt \qquad a \le x \le b$$

is continuous on $[a, b]$ and differentiable on $(a, b)$, and

$$g'(x) = f(x).$$

## FTC1 IN LEIBNIZ NOTATION

**Theorem**: If $f$ is continuous on $[a, b]$, then the function $g$ defined by

$$g(x) = \int_a^x f(t)dt \qquad a \le x \le b$$

is continuous on $[a, b]$ and differentiable on $(a, b)$, and

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

## THE FUNDAMENTAL THEOREM OF CALCULUS, PART 2 (FTC2)

**Theorem**: If $f$ is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F$ is any antiderivative of $f$, that is, a function such that $F' = f$.

## NOTATIONS FOR *F*(*b*) − *F*(*a*)

$$\begin{aligned} F(b) - F(a) &= F(x)]_a^b \\ &= F(x)\Big|_a^b \\ &= [F(x)]_a^b \end{aligned}$$

## THE FUNDAMENTAL THEOREM OF CALCULUS

**Theorem:** Suppose $f$ is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$.
2. $\int_a^b f(x)dx = F(b) - F(a)$, where $F$ is any antiderivative of $f$, that is, $F' = f$.