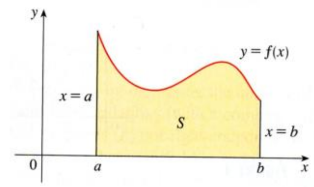
Section 4.1

Areas and Distances

THE AREA PROBLEM

The **area problem** is to find the area of the region S that lies under the curve $y = f(x)$ from a to b. See the figure below.



FINDING THE AREA UNDER A CURVE

1. Divide (partition) the interval $[a, b]$ into n equal pieces (subintervals) of width $\Delta x = \frac{b-a}{n}$
2. The subintervals are: $[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$. Note that

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

AREA (CONTINUED)

3. Add up area of right (left) rectangles.

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x$$

4. Take the limit as n approaches infinity to find true area under the curve.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n$$

AREA

Instead of using left endpoints or right endpoints, we could take the height of the rectangle to be the value of f at <u>any</u> number x_i^* in the ith subinterval $[x_{i-1}, x_i]$. These numbers are called **sample points**. Thus, the area can be given by

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

See Figure 13 on page 299.

THE DISTANCE PROBLEM

The **distance problem** is to find the distance traveled by an object during a certain time period if the velocity of the object is known at all times.

EXAMPLE

Speedometer readings for a motorcycle at 12-second intervals are given in the table below. Find two estimates for the distance traveled by the motorcycle for this 60-second period.

t (sec)	0	12	24	36	48	60
v (ft/sec)	30	28	25	22	24	27

DISTANCE TRAVELED

The distance, d, traveled by an object with velocity $v = f(t)$ is

$$d = \lim_{n\to\infty} \sum_{i=1}^{n} f(t_{i-1})\Delta t = \lim_{n\to\infty} \sum_{i=1}^{n} f(t_i)\Delta t$$