Section 3.9

Antiderivatives

ANTIDERIVATIVES

Definition: A function F is called an **antiderivative** of f on the interval I if $F'(x) = f(x)$ for all x in the interval I.

A NOTATION FOR THE ANTIDERIVATIVE

If the function $F(x)$ is an antiderivative of the function $f(x)$, we can use the following notation:

$$A_x f(x) = F(x)$$

For Example: Since $F(x) = x^2$ is an antiderivative of $f(x) = 2x$, we can write

$$A_x(2x) = x^2$$

THE "GENERAL" ANTIDERIVATIVE

Theorem: If F is an antiderivative of f on the interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

For Example: The most general antiderivative of $2x$ is $x^2 + C$; that is,

$$A_x(2x) = x^2 + C$$

PARTICULAR ANTIDERIVATIVES

Definition: A **particular antiderivative** is an antiderivative with a specific number for the constant, C, in the general antiderivative.

For Example: Each of the following is a particular antiderivative of $2x$.

x^2

$x^2 + 4$

$x^2 - 98$

SOME ANTIDERIVATIVE FORMULAS

Power Rule:

$$A_x(x^n) = \frac{x^{n+1}}{n+1} + C, n \neq -1$$
$$= \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

Addition/Subtraction Rule:

$$A_x[f(x) \pm g(x)] = A_x[f(x)] \pm A_x[g(x)]$$

Constant Multiple Rule:

$$A_x[k \cdot f(x)] = k \cdot A_x[f(x)]$$

SOME TRIGONOMETRIC ANTIDERIVATIVES

$$A_x(\cos x) = \sin x + C$$

$$A_x(\sin x) = -\cos x + C$$

$$A_x(\sec^2 x) = \tan x + C$$

$$A_x(\sec x \tan x) = \sec x + C$$

$$A_x(\csc^2 x) = -\cot x + C$$

$$A_x(\csc x \cot x) = -\csc x + C$$

DIFFERENTIAL EQUATION

Definition: An equation that involves the derivatives of a function is called a **differential equation**.