Section 3.7

Optimization Problems

FIRST DERIVATIVE TEST FOR ABSOLUTE EXTREME VALUES

Suppose that c is a critical number of a continuous function defined on an interval.

(a) If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f.

(b) If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f.

PROCEDURE FOR SOLVING OPTIMIZATION PROBLEMS

1. Draw and label a picture.
2. Write a formula for quantity to be optimized.
3. Use conditions in the problem to eliminate all but one variable.
4. Determine domain.
5. Find critical numbers and endpoints if necessary.
6. Use tests to determine which point yields the optimum solution.

EXAMPLES

1. A rectangular box is to be made from a sheet of tin 20 inches square by cutting a square from each corner and turning up the sides. Find the maximum volume of such a box.

2. A soup can is to hold 54π cubic centimeters. If the cost of the can is proportional to the amount of material used (*i.e.*, the surface area of the can), find the dimensions which minimize the cost of the can.

EXAMPLES (CONTINUED)

3. Agent 00π is waiting at the edge of a straight canal 1 mile wide, in a motor boat capable of going 40 mph. There is a straight road along the opposite edge of the canal; her partner will have a 50 mph motorcycle waiting for her wherever she lands. At midnight she will receive a package to be delivered to a man in a Mercedes-Benz 5 miles down the road. (See figure.)
Mission: Deliver the package in the shortest possible time.
Problem: Where should her partner park the motorcycle?


FIGURE FOR EXAMPLE 3
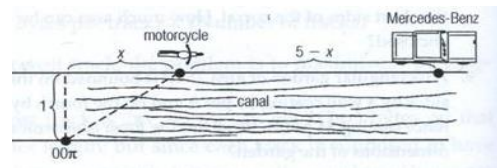
EXAMPLES (CONTINUED)

4. Computer disk packs (see figure below) contain information in units called *bytes* that are arranged in concentric circular tracks on each disk in the pack. (A typical disk is also shown below.) Manufacturing constraints limit the density to 100 bytes per centimeter along a given track, and 50 tracks per centimeter measure radially across the disk. Assume that each disk has an outside radius of 5 cm. If the number of bytes on each track must be the same (to achieve uniformity in reading the information), where should the innermost track be located to get the maximum number of bytes on the disk? What is the maximum number of bytes on a disk?

FIGURES FOR EXAMPLE 4
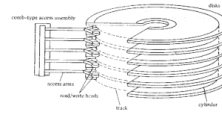
Structure of a disk pack.
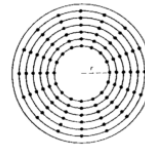
Bytes on a computer disk.

EXAMPLES (CONCLUDED)

5. A cistern with a square base is to be constructed to hold 12,000 cubic feet of water. If the metal top cost twice as much per square foot as the concrete sides and base, what are the most economical dimensions for the cistern?