Section 3.5

Summary of Curve Sketching

THINGS TO CONSIDER BEFORE SKETCHING A CURVE

- Domain
- Intercepts
- Symmetry - even, odd, periodic.
- Asymptotes - vertical, horizontal, slant.
- Intervals of increase or decrease.
- Local maximum or minimum values.
- Concavity and Points of Inflection

Not every item above is relevant to every function.

PROCEDURE FOR CURVE SKETCHING

Step 1: Precalculus analysis

(a) Check the *domain* of the function to see if any regions of the plane are excluded.

(b) Find the x- and y-*intercepts*.

(c) Test for *symmetry* with respect to the y-axis and the origin. (Is the function even or odd?)

PROCEDURE (CONTINUED)

Step 2: Calculus Analysis

(a) Find the *asymptotes* (vertical, horizontal, and/or slant).

(b) Use the first derivative to find the critical points and to find the intervals where the graph is *increasing* and *decreasing*.

(c) Test the critical points for *local maxima* and *local minima*.

(d) Use the second derivative to find the intervals where the graph is *concave up* and *concave down* and to locate *inflection points*.

PROCEDURE (CONCLUDED)

Step 3: Plot a few points (including ***all*** critical points, inflection points, and intercepts).

Step 4: Sketch the graph. (NOTE: On the graph, label ***all*** critical points, inflection points, intercepts and asymptotes.)

SLANT ASYMPTOTES

The line $y = mx + b$ is a **slant** (or **oblique**) **asymptote** of the graph of $y = f(x)$ if

$$\lim_{x\to\pm\infty} [f(x) - (mx + b)] = 0$$

or

$$\lim_{x\to\pm\infty} f(x) = mx + b$$

SLANT ASYMPTOTES AND RATIONAL FUNCTIONS

For rational functions, slant asymptotes occur when the degree of the numerator is ***exactly*** one higher than the degree of the denominator.

For rational functions, the slant asymptote can be found by using ***long division of polynomials***.