Section 3.3

How Derivatives Affect the Shape of a Graph

INCREASING/DECREASING TEST

<u>Theorem</u>:

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

THE FIRST DERIVATIVE TEST

Suppose that c is a critical number of a continuous function f.

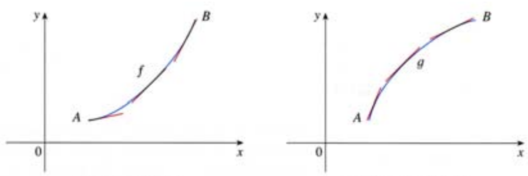
(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change signs at c (that is, f' is positive or negative on both sides of c), then f has no local minimum or maximum at c.

CONCAVITY

<u>Definition</u>: If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward**.



(a) Concave upward (b) Concave downward

CONCAVITY TEST

<u>Theorem</u>:

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

INFLECTION POINT

A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous at P and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

THE SECOND DERIVATIVE TEST

Theorem: Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

COMMENTS ON THE SECOND DERIVATIVE TEST

Suppose c is a critical number of the function $y = f(x)$.

- If $f''(c) = 0$, then the Second Derivative Test is inclusive. That is, the ***First Derivative Test must be used*** to determine local minimum or maximum. EXAMPLE: $f(x) = x^4$
- If $f'(c)$ is undefined (does not exist), then the Second Derivative Test is inclusive. That is, the ***First Derivative Test must be used*** to determine local minimum or maximum. EXAMPLE: $f(x) = x^{2/3}$

EXAMPLES

1. Find the local minima and local maxima (if any) of $$f(x) = \sqrt[3]{x}\,(8 - x).$$

2. Sketch the graph of a function that satisfies all the conditions:
   $f'(-1) = f'(1) = 0$
   $f'(x) < 0$ if $-1 < x < 1$
   $f'(x) > 0$ if $1 < x < 2$ or $-2 < x < -1$
   $f'(x) = -1$ if $x < -2$ or $x > 2$
   $f''(x) < 0$ if $-2 < x < 0$
   The point $(0,1)$ is an inflection point