

Section 3.2

The Mean Value Theorem

ROLLE'S THEOREM

Theorem: Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b) .
3. $f(a) = f(b)$.

Then there is a number c in (a, b) such that $f'(c) = 0$.

THE MEAN VALUE THEOREM (MVT)

Theorem: Let f be a function that satisfies the following two hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b) .

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a)$$

TWO THEOREMS

Theorem: If $f'(x) = 0$ for all x in an interval (a, b) , then f is a constant on (a, b) .

Theorem: If $f'(x) = g'(x)$ for all x in an interval (a, b) , then $f - g$ is a constant on (a, b) ; that is $f(x) = g(x) + c$, where c is a constant.