# Section 3.1

**Maximum and Minimum Values**

## ABSOLUTE MINIMUM AND ABSOLUTE MAXIMUM

1. A function $f$ has an **absolute** (or **global**) **maximum** at $c$ if $f(c) \geq f(x)$ for all $x$ in $D$, where $D$ is the domain of $f$. The number $f(c)$ is called the maximum value of $f$ on $D$.
2. A function $f$ has an **absolute** (or **global**) **minimum** at $c$ if $f(c) \leq f(x)$ for all $x$ in $D$, where $D$ is the domain of $f$. The number $f(c)$ is called the minimum value of $f$ on $D$.
3. The maximum and minimum values of $f$ are called the **extreme values** (also called **extrema**)of $f$.

## LOCAL MINIMUM AND LOCAL MAXIMUM

1. A function $f$ has a **local** (or **relative**) **maximum** at $c$ if $f(c) \geq f(x)$ for all $x$ near $c$.
2. A function $f$ has a **local** (or **relative**) **minimum** at $c$ if $f(c) \leq f(x)$ for all $x$ near $c$.

## THE EXTREME VALUE THEOREM

**Theorem:** If $f$ is continuous on a closed interval $[a, b]$, then $f$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers $c$ and $d$ in $[a, b]$.

## FERMAT'S THEOREM

**Theorem:** If $f$ has a local maximum or minimum at $c$, and if $f'(c)$ exists, then $f'(c) = 0$.

## CRITICAL NUMBERS

A **critical number** of a function $f$ is a number $c$ in the domain of $f$ such that either $f'(c) = 0$ or $f'(c)$ does not exist.

## FERMAT'S THEOREM REVISITED

Fermat's Theorem can be stated using the idea of critical numbers as follows.

**Fermat's Theorem:** If $f$ has a local minimum or local maximum at $c$, then $c$ is a critical number of $f$.

## EXTREME VALUE THEOREM RESTATED

If $f$ is *continuous* on a *closed interval* $[a, b]$, then $f$ attains both an absolute minimum and absolute maximum value on that interval.

## THE CLOSED INTERVAL METHOD

To find the *absolute* maximum and minimum values of a continuous function on a closed interval $[a, b]$:

1. Find the values of $f$ at the critical numbers of $f$ in $(a, b)$.
2. Find the values of $f$ at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.