Section 2.6

Implicit Differentiation

EXPLICIT AND IMPLICIT FUNCTIONS

Definition: The function y is a **explicit function** of x if the formula is written in the form

$$y = \text{expression with } x\text{'s.}$$

If the formula is NOT written this way, we say that y is an **implicit function** of x.

EXAMPLE

Below is the same function defined explicitly and implicitly.

EXPLICIT: $y = x^{2/3}$

IMPLICIT: $y^3 = x^2$

IMPLICIT DIFFERENTIATION

Many times if a function is defined implicitly, it is difficult (or impossible) to write it explicitly. To take the derivative of an implicitly defined function, we use **implicit differentiation**.

PROCEDURE FOR IMPLICIT DIFFERENTIATION

1. Assuming y is a function of x, differentiate both sides with respect to x using the chain rule for y terms.

2. Collect the terms with $\frac{dy}{dx}$ on one side and the other terms on the other side.

3. Factor out $\frac{dy}{dx}$.

4. Divide both sides by the factor to leave $\frac{dy}{dx}$ by itself.