Section 2.5

The Chain Rule

THE CHAIN RULE

Theorem: If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f\big(g(x)\big)$ is differentiable at x and F' is given by the product

$$F'(x) = f'\big(g(x)\big) \cdot g'(x).$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

"INTUITIVE" CHAIN RULE

In using the Chain Rule, we work from the outside to the inside. The formula says that ***we differentiate the outer function f [at the inner function $g(x)$] and then we multiply by the derivative of the inner function.***

THE POWER RULE COMBINED WITH THE CHAIN RULE

Theorem: If n is any real number and $u = g(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1} \cdot \frac{du}{dx}$$

Alternatively,

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$