

#### THE TANGENT LINE TO A CURVE AT A POINT

**Definition:** The **tangent line** to the curve y = f(x) at the point P(a, f(a)) is the line through *P* with slope *m* where

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$
$$= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

provided the limit exists.

We sometimes refer to the slope of the tangent line at a point as the <u>slope of the curve</u> at that point.

# INSTANTANEOUS VELOCITY

Suppose s = f(t) is the position function of a moving object. The <u>velocity</u> (or <u>instantaneous</u> <u>velocity</u>), v(a), at time t = a is

$$v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

# THE DERIVATIVE AT A POINT

**Definition:** The derivative of a function *f* at number *a*, denoted by f'(a), is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

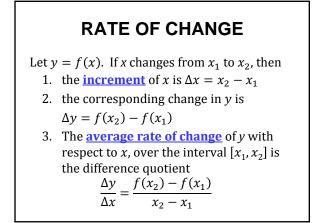
if the limit exists.

# AN ALTERNATIVE FORM OF THE DERIVATIVE

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

### INTERPRETATIONS OF THE DERIVATIVE

- 1. The tangent line to y = f(x) at (a, f(a)) is the line through (a, f(a)) whose slope is equal to f'(a), the derivative of f at a.
- 2. The instantaneous velocity at time *a* of a particle whose position is described by s = f(t) is give by f'(a), the derivative of *f* at *a*.



### RATE OF CHANGE (CONCLUDED)

4. The **instantaneous rate of change** is

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

## ANOTHER INTERPRETATION OF THE DERIVATIVE

The derivative f'(a) is the instantaneous rate of change of y = f(x) with respect to x when x = a.