Section 2.1

Derivatives and Rates of Change

THE TANGENT LINE TO A CURVE AT A POINT

Definition: The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope m where

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

We sometimes refer to the slope of the tangent line at a point as the **slope of the curve** at that point.

INSTANTANEOUS VELOCITY

Suppose $s = f(t)$ is the position function of a moving object. The **velocity** (or **instantaneous velocity**), $v(a)$, at time $t = a$ is

$$v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

THE DERIVATIVE AT A POINT

Definition: The **derivative of a function f at number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if the limit exists.

AN ALTERNATIVE FORM OF THE DERIVATIVE

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

INTERPRETATIONS OF THE DERIVATIVE

1. The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.
2. The instantaneous velocity at time a of a particle whose position is described by $s = f(t)$ is give by $f'(a)$, the derivative of f at a.

RATE OF CHANGE

Let $y = f(x)$. If x changes from x_1 to x_2, then

1. the **increment** of x is $\Delta x = x_2 - x_1$
2. the corresponding change in y is $\Delta y = f(x_2) - f(x_1)$
3. The **average rate of change** of y with respect to x, over the interval $[x_1, x_2]$ is the difference quotient
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

RATE OF CHANGE (CONCLUDED)

4. The **instantaneous rate of change** is
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

ANOTHER INTERPRETATION OF THE DERIVATIVE

The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$.