Section 1.8

Continuity

CONTINUITY AT A NUMBER *a*

Definition: A function f is **continuous at a number a** if

$$\lim_{x\to a} f(x) = f(a).$$

INTERPRETATION OF THE DEFINITION

Notice that the definition on the previous slide implicitly requires three things if f is continuous at a.

1. $f(a)$ is defined (that is, a is in the domain of f)
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x) = f(a)$

DISCONTINUITIES

Definition: If f is defined near a, we say that f has a **discontinuity** at a, or f is **discontinuous at a**, if f is not continuous at a.

TYPES OF DISCONTINUITIES

There are three types of discontinuities.

1. **Removable Discontinuity at a:** A discontinuity that can be removed by redefining f at the number a.
2. **Infinite Discontinuity at a:** A discontinuity where the limit of f as x approaches a is either ∞ or $-\infty$.
3. **Jump Discontinuity at a:** A discontinuity where the left-hand and right-hand limits are different at a (and neither limit is ∞ or $-\infty$).

LEFT CONTINUOUS AND RIGHT CONTINUOUS

Definition: A function f is **continuous from the left at a number a** if

$$\lim_{x\to a^-} f(x) = f(a)$$

Definition: A function f is **continuous from the right at a number a** if

$$\lim_{x\to a^+} f(x) = f(a)$$

CONTINUITY ON AN INTERVAL

Definition: A function f is **continuous on an interval** if it is continuous at every number in the interval.

THEOREM

If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$ 2. $f - g$ 3. cf

4. fg 5. $\frac{f}{g}$ if $g(a) \neq 0$

CONTINUITY OF POLYNOMIALS AND RATIONAL FUNCTIONS

Theorem:

a) Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.

b) Any rational function is continuous wherever it is defined; that is, it is continuous in its domain.

The following types of functions are continuous at every number in their domains:

polynomials	rational functions
root functions	trigonometric functions

THE COMPOSITE LIMIT THEOREM

Theorem: If f is continuous at b and $\lim_{x\to a} g(x) = b$, then $\lim_{x\to a} f(g(x)) = f(b)$.

In other words,

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right).$$

This theorem says that a limit symbol can be moved through a function symbol if the function is continuous and the limit exists. In other words, the order of these two symbols can be reversed.

A COROLLARY OF THE COMPOSITE LIMIT THEOREM

If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

THE INTERMEDIATE VALUE THEOREM

<u>Theorem</u>: Suppose that f is continuous on the closed interval $[a, b]$ and let N be a number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$.. Then there exists a number c in (a, b) such that $f(c) = N$.