Section 1.7

The Precise Definition of a Limit

THE PRECISE DEFINITION OF A LIMIT

Definition: Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say the **limit of $f(x)$ as x approaches a is L**, and we write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \quad \text{whenever} \quad 0 < |x - a| < \delta.$$

Alternatively,

$$\text{if} \quad 0 < |x - a| < \delta, \text{ then } |f(x) - L| < \varepsilon.$$

LEFT-HAND AND RIGHT-HAND LIMITS

See page 77 of the text for the precise definition of left-hand and right-hand limits.

INFINITE LIMITS

Definition: Let f be a function defined on some open interval that contains the number a, except possibly at a. Then

$$\lim_{x \to a} f(x) = \infty$$

means that for every positive number M there is a positive number δ such that

$$f(x) > M \quad \text{whenever} \quad 0 < |x - a| < \delta.$$

There is a similar definition for

$$\lim_{x \to a} f(x) = -\infty$$

on page 80 of the text.