Section 1.6

Calculating Limits Using the Limit Laws

LIMIT LAWS THEOREM

Suppose that c is a constant and the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a} [c \cdot f(x)] = c \cdot \lim_{x \to a} f(x)$
4. $\lim_{x \to a} [f(x) \cdot g(x)] = \left[\lim_{x \to a} f(x)\right] \cdot \left[\lim_{x \to a} g(x)\right]$
5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

LIMIT LAWS (CONTINUED)

6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer
7. $\lim_{x \to a} c = c$
8. $\lim_{x \to a} x = a$
9. $\lim_{x \to a} x^n = a^n$
10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positve integer

    (If n is even, we assume that $a > 0$.)
11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positve integer

    (If n is even, we assume that $\lim_{x \to a} f(x) > 0$.)

DIRECT SUBSTITUTION PROPERTY

If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a).$$

ANOTHER LIMIT PROPERTY

If $f(x) = g(x)$ when $x \neq a$, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$$

provided the limit exists.

A LIMIT THEOREM

Theorem: If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x).$$

THE SQUEEZE THEOREM

Theorem: If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$

NOTE: This theorem is also sometimes called the *Sandwich Theorem*.