Section 1.6
Calculating Limits Using the Limit Laws

**LIMIT LAWS THEOREM**
Suppose that \( c \) is a constant and the limits \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) exist. Then
1. \( \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) \)
2. \( \lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) \)
3. \( \lim_{x \to a} [c \cdot f(x)] = c \cdot \lim_{x \to a} f(x) \)
4. \( \lim_{x \to a} [f(x) \cdot g(x)] = \left( \lim_{x \to a} f(x) \right) \cdot \left( \lim_{x \to a} g(x) \right) \)
5. \( \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \) if \( \lim_{x \to a} g(x) \neq 0 \)

**LIMIT LAWS (CONTINUED)**
6. \( \lim_{x \to a} [f(x)]^n = \left( \lim_{x \to a} f(x) \right)^n \) where \( n \) is a positive integer
7. \( \lim_{x \to a} c = c \)
8. \( \lim_{x \to a} x = a \)
9. \( \lim_{x \to a} x^n = a^n \)
10. \( \lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a} \) where \( n \) is a positive integer
   (If \( n \) is even, we assume that \( a > 0 \).)
11. \( \lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} \) where \( n \) is a positive integer
   (If \( n \) is even, we assume that \( \lim_{x \to a} f(x) > 0 \).)

**DIRECT SUBSTITUTION PROPERTY**
If \( f \) is a polynomial or a rational function and \( a \) is in the domain of \( f \), then
\[ \lim_{x \to a} f(x) = f(a). \]

**ANOTHER LIMIT PROPERTY**
If \( f(x) = g(x) \) when \( x \neq a \), then
\[ \lim_{x \to a} f(x) = \lim_{x \to a} g(x) \]
provided the limit exists.

**A LIMIT THEOREM**
**Theorem:** If \( f(x) \leq g(x) \) when \( x \) is near \( a \) (except possibly at \( a \)) and the limits of \( f \) and \( g \) both exist as \( x \) approaches \( a \), then
\[ \lim_{x \to a} f(x) \leq \lim_{x \to a} g(x). \]
THE SQUEEZE THEOREM

**Theorem:** If \( f(x) \leq g(x) \leq h(x) \) when \( x \) is near \( a \) (except possibly at \( a \)) and

\[
\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,
\]

then

\[
\lim_{x \to a} g(x) = L.
\]

**NOTE:** This theorem is also sometimes called the *Sandwich Theorem.*