Section 1.5

The Limit of a Function

INTUITIVE DEFINITION OF A LIMIT OF A FUNCTION

Definition: Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

and say

"the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking values of x to be sufficiently close to a (on either side of a) but not equal to a.

ALTERNATIVE NOTATION FOR THE LIMIT

An alternative notation for

$$\lim_{x \to a} f(x) = L$$

is

$$f(x) \to L \text{ as } x \to a$$

LEFT-HAND LIMITS

Definition: We write

$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$, as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking values of x to be sufficiently close to a with x ***less than*** a, we

RIGHT-HAND LIMITS

Definition: We write

$$\lim_{x \to a^+} f(x) = L$$

and say the **right-hand limit of $f(x)$, as x approaches a** [or the **limit of $f(x)$ as x approaches a from the right**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking values of x to be sufficiently close to a with x ***greater than*** a, we

A LEFT-HAND RIGHT-HAND LIMIT THEOREM

Theorem:

$$\lim_{x \to a} f(x) = L$$

if and only if

$$\lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L$$

INFINITE LIMITS

Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made as positively large as we please by taking values of x sufficiently close to a, but not equal to a.

Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that the values of $f(x)$ can be made as negatively large as we please by taking values of x sufficiently close to a, but not equal to a.

VERTICAL ASYMPTOTES

Definition: The line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

It is ***NOT*** the vertical asymptotes that cause the limits to be ∞ or $-\infty$, but rather the limits being ∞ or $-\infty$ that create the vertical asymptotes.