Section 1.4

The Tangent and Velocity Problems

WHAT IS A TANGENT LINE TO THE GRAPH OF A FUNCTION?

A line l is said to be a tangent to a curve at a point P if the line l touches, but does not intersect, the curve at P.

SECANT LINES

A **secant line** to a curve is a line that goes through two points, P and Q, on the curve. We will denote the **slope** of the secant line through PQ by m_{PQ}.

In particular, if $y = f(x)$ is our function and P is the point $(a, f(a))$ and Q is the point $(x, f(x))$, then the slope of the secant line is given by

$$m_{PQ} = \frac{f(x) - f(a)}{x - a}$$

THE SLOPE OF THE TANGENT LINE

The **slope** (m) of the tangent line at P is the **limit** of the slopes of the secant lines PQ as Q approaches P. That is,

$$m = \lim_{Q \to P} m_{PQ}$$

In particular, if $y = f(x)$ is our function and P is the point $(a, f(a))$ and Q is the point $(x, f(x))$, then the slope of the tangent line at $(a, f(a))$ is given by

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

AVERAGE VELOCITY

The **average velocity** of an object is the distance the object traveled divided by the elapsed time. That is,

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

In particular, if $s = f(t)$ describes the position of moving object at time t, then the average velocity of the object between time t and time a is

$$\text{avg. vel.} = \frac{f(t) - f(a)}{t - a}$$

INSTANTANEOUS VELOCITY

The **instantaneous velocity** is the limiting value of the average velocities over shorter and shorter time periods.

In particular, if $s = f(t)$ describes the position of moving object at time t, then the instantaneous velocity of the object between at time a is

$$v = \lim_{t \to a} \frac{f(t) - f(a)}{t - a}$$