STUDY GUIDE FOR TEST II
MATH 1501

1. Use the ***definition of derivative*** to find the derivative of the following functions. The definition of the derivative is
$$f'(x) = \lim_{x\to 0}\frac{f(x+h)-f(x)}{h}.$$

(a) $f(x) = \sqrt{x^2+1}$

(b) $f(x) = \frac{1}{x^2+1}$

2. Find the derivative for each of the following functions.

(a) $y = 12x^3 - x^2 + 3x$

(b) $g(x) = \frac{2x^2-3x+1}{x}$

(c) $y = 10\tan t + \frac{1}{t^2}$

(d) $f(x) = 7x^4 - 6x^3 + 5x^2 + 1$

(e) $y = (8+2x)(x^2+1)^4$

(f) $h(u) = \frac{u^2-1}{u^2+1}$

(g) $y = \cos^3 t$

(h) $y = \sqrt[3]{x} + \sqrt[5]{x}$

(i) $g(x) = \sqrt[3]{4x-1}$

(j) $y = 5x\sqrt{1-2x}$

(k) $y = \sqrt{x^4 + \tan x}$

(l) $f(x) = \sin\left(\sqrt{x^2+2x+3}\right)$

3. Find an equation of the tangent line to curve at the indicated point.

(a) $f(x) = x^2 - 6x + 4;\ (2,-4)$

(b) $f(x) = \frac{4x+5}{x^2};\ (-1,1)$

(c) $y = \sin x + 2\cos x;\ \left(\frac{\pi}{2},1\right)$

(d) $y = \tan\frac{x}{2} + 2;\ \left(\frac{\pi}{2},3\right)$

4. (a) Find the x-coordinates of all points on the graph of the function
$$y = x^3 - 3x$$
at which the tangent line is horizontal.

(b) Find the x-coordinates of all points on the graph of the function
$$f(x) = x^3 - x^2 + x + 5$$
at which the tangent line has slope 2.

5. Find the indicated derivative.

(a) $y = 2x^3 - 6x^2 - 7x - 2;\ \ y'''$

(b) $y = (2x+1)^4;\ \frac{d^2y}{dx^2}$

(c) $y = 3\cos x + x^{-3};\ \frac{d^3y}{dx^3}$

6. Use Implicit Differentiation to find $\frac{dy}{dx}$ for the following functions.

(a) $4x^2 - 3x^2y + 19xy = 0$

(b) $\sqrt{xy} + 3y = 10x$

(c) $x\sin(xy) = x^2 + 1$

7. Given $f(2) = 3,\ f'(2) = -2, g(2) = 5, g'(2) = 4$, and $f'(5) = -6$.

(a) Find $(f-g)'(2)$.

(b) Find $(fg)'(2)$.

(c) Find $\left(\frac{f}{g}\right)'(2)$.

(d) If $H(x) = f\big(g(x)\big)$, find $H'(2)$.

8. Solve the following related rate problems.

(a) A ladder 41 feet long that was leaning against a vertical wall begins to slip. Its top slides down the wall while its bottom moves along the level ground at the constant speed of 4 feet per second. How fast is the top of the ladder moving when it is 9 feet above the ground?

(b) Water is flowing into a conical tank (vertex down) of height 10 meters and radius 6 meters in such a way that the water level is rising at the rate of $\frac{1}{2}$ meters per minute. How fast is the volume of water in the tank increasing when the water in the tank is 5 meters deep?

(c) A cubical box with sides of length 15 inches is covered on all sides with ice of uniform thickness. The ice is melting at the rate of 80 cubic inches per hour. How fast is the thickness of the ice changing when the ice is 1 inch thick?

(d) A spherical balloon is inflated with gas at the rate of 500 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is 30 centimeters?

ANSWERS

1. (a) $f'(x) = \frac{x}{\sqrt{x^2+1}}$ (b) $\frac{dy}{dx} = -\frac{2x}{(x^2+1)^2}$

2. (a) $\frac{dy}{dx} = 36x^2 - 2x + 3$ (b) $g'(x) = 2 - \frac{1}{x^2}$

   (c) $\frac{dy}{dx} = 10\sec^2 t - \frac{2}{t^3}$ (d) $f'(x) = 28x^3 - 18x^2 + 10x$

   (e) $\frac{dy}{dx} = 2(x^2+1)^3(9x^2 + 32x + 1)$ (f) $h'(u) = \frac{4u}{(u^2+1)^2}$

   (g) $\frac{dy}{dx} = -3\cos^2 t \sin t$ (h) $\frac{dy}{dx} = \frac{1}{3x^{2/3}} + \frac{1}{5x^{4/5}}$

   (i) $g'(x) = \frac{4}{3(4x-1)^{2/3}}$ (j) $\frac{dy}{dx} = \frac{5(1-3x)}{\sqrt{1-2x}}$

   (k) $\frac{dy}{dx} = \frac{4x^3 + \sec^2 x}{2\sqrt{x^4 + \tan x}}$ (l) $f'(x) = \frac{(x+1)\cos\left(\sqrt{x^2+2x+3}\right)}{\sqrt{x^2+2x+3}}$

3. (a) $y = -2x$ (b) $y = 6x + 7$

   (c) $y = -2x + (\pi + 1)$ (d) $y = x + \left(3 - \frac{\pi}{2}\right)$

4 (a) $x = 1, -1$ (b) $x = -\frac{1}{3}, 1$

5. (a) $y''' = 12$ (b) $\frac{d^2y}{dx^2} = 48(2x+1)^2$

   (c) $\frac{d^3y}{dx^3} = 3\sin x - 60x^{-6}$

6. (a) $\frac{dy}{dx} = \frac{6xy - 19y - 8x}{19x - 3x^2}$ (b) $\frac{dy}{dx} = \frac{20\sqrt{xy} - y}{x + 6\sqrt{xy}}$

   (c) $\frac{dy}{dx} = \frac{2x - xy\cos(xy) - \sin(xy)}{x^2\cos(xy)}$

7. (a) -6 (b) 2

   (c) $-\frac{22}{25}$ (d) -24

8. (a) $-\frac{160}{9}$ ft/sec (b) $\frac{9\pi}{2}$ m^3/min

   (c) $-\frac{40}{867}$ in/hr (d) $\frac{5}{36\pi}$ cm/min