REVIEW FOR TEST I
MATH 1501

1. (a) Complete the following table for $f(x) = \frac{\ln(x+5) - \ln 5}{x}$.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

Based on the evidence above, $\lim_{x\to 0} \frac{f(x+5)-f(5)}{x} = \lim_{x\to 0} \frac{\ln(x+5)-\ln 5}{x} =$ ____.

(b) Use the table below to find the following limits.

x	2	2.5	2.9	2.99	$\rightarrow$	3	$\leftarrow$	3.01	3.1	3.5	4
$f(x)$	7	6.25	6.125	6.0625	$\rightarrow$	?	$\leftarrow$	9.1026	9.283	9.67	10

i. $\lim_{x\to 3^-} f(x)$ ii. $\lim_{x\to 3^+} f(x)$

iii. $\lim_{x\to 3} f(x)$

2. Find the indicated limit or state it does not exist. If it does not exist, tell why. Use $-\infty$ and ∞ as appropriate.

(a) $\lim_{x\to 1} \frac{x^2-1}{x+1}$

(b) $\lim_{x\to 0} \frac{\cos x}{x}$

(c) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$

(**Hint:** Rationalize the denominator.)

(d) $\lim_{x\to \frac{1}{2}^-} [\![4x]\!]$

(**Note:** $[\![x]\!]$ is the greatest integer function.)

(e) $\lim_{x\to 1^-} f(x)$ where $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 < x < 1 \\ 1-x & \text{if } x \geq 1 \end{cases}$

(f) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 < x < 1 \\ 1-x & \text{if } x \geq 1 \end{cases}$

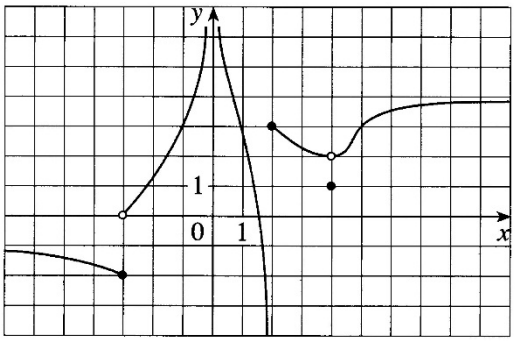
(g) $\lim_{x\to -1} (2x^2 + 6x - 1)$

(h) $\lim_{x\to 3} \frac{x^2-9}{x^2-3x}$

(i) $\lim_{x\to 3^-} \frac{x+1}{x^2-9}$

(j) $\lim_{x\to 3} \frac{x+1}{x^2-9}$

3. Use the graph of $y = f(x)$ below to answer the following. If a limit does not exist, explain why.



(a) $\lim_{x\to 2^+} f(x)$

(b) $\lim_{x\to -3^+} f(x)$

(c) $\lim_{x\to -3} f(x)$

(d) $\lim_{x\to 4} f(x)$

(e) $\lim_{x\to 0} f(x)$

(f) $\lim_{x\to 2^-} f(x)$

(g) State the equation(s) of any vertical asymptotes.

(h) At what numbers is f discontinuous? Explain using the definition of continuity. Also, at what numbers (if any) is f right- or left-continuous?

4. Determine at what points, if any, the following functions are discontinuous. Classify any points of discontinuity as removable or nonremovable.

   (a) $f(x) = 4x^2 - 2x + 12$

   (b) $g(x) = \frac{x^3-8}{x-2}$

   (c) $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 2 \\ 2 - x & \text{if } x > 2 \end{cases}$

   (d) $f(x) = \frac{1}{x-1}$

   (e) $f(x) = \frac{x}{x^2+1}$

   (f) $f(x) = \frac{x-1}{x^2+x-2}$

5. Use the Intermediate Value Theorem to show that the equation $x^5 - 4x^3 - 3x + 1 = 0$ has at least one solution between $x = 2$ and $x = 3$.

6. Sketch the graph of a function that satisfies all the following conditions.
   - Its domain is $[0, 6]$
   - $f(0) = f(2) = f(4) = f(6) = 2$
   - f is continuous except at $x = 2$.
   - $\lim_{x\to 2^-} f(x) = 1$ and $\lim_{x\to 2^+} f(x) = 2$

7. Identify any vertical asymptotes of the graphs of the following functions. Indicate the limit from each side of a vertical asymptote.

   (a) $f(x) = \frac{1}{(x+3)^4}$

   (b) $f(x) = \frac{4x^2}{4-x^2}$

   (c) $f(x) = \frac{-4x}{x^2+4}$

8. (a) The point $P(1, -3)$ lies on the curve $f(x) = x^2 - 5x + 1$. Find the slope of the secant line PQ where Q is the point on the graph where $x = 2$.

   (b) A ball is thrown straight upward with an initial velocity of 48 ft/sec

from the edge of a building 196 feet tall. Its height, in feet, above the ground after t seconds is given by $f(t) = 196 + 48t - 16t^2$. Using the definition, find the **average velocity** over the time interval $[3, 5]$.

9. (a) The table below gives the values of a function f at certain values of x. Estimate the slope of the **tangent line** at $x = 0$.

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	$\frac{1}{2}$	$\frac{4}{7}$	$\frac{2}{3}$	$\frac{4}{5}$	1

(b) This displacement s, measured in meters, of an object moving in a straight line at time t, measured in seconds, is given by the table below. Estimate the **instantaneous velocity** at $t = 6$ seconds.

t (seconds)	0	2	4	6	8
s (meters)	1	6	13	22	33

ANSWERS

1. (a) 0.2
   (b) i. 6
   ii. 9
   iii. DNE, different limits from each side

2 (a) 0
   (b) DNE, two different directions
   (c) 4
   (d) 1
   (e) 1
   (f) DNE, different limits from each side
   (g) -5
   (h) 2
   (i) $-\infty$

(j) DNE, different limits from each side

3. (a) 3
   (b) 0
   (c) DNE, different limits from each side
   (d) 2
   (e) ∞
   (f) $-\infty$
   (g) $x = 0; x = 2$
   (h) The function f is discontinuous at -3 (because limit DNE); at 0 (because limit DNE); at 2 (limit DNE); and at 4 (limit not equal to function value). The function is left-continuous at -3 and right-continuous at 2.

4. (a) continuous everywhere
   (b) discontinuous at $x = 2$ (removable)
   (c) discontinuous at $x = 2$ (nonremovable)
   (d) $x = 1$ (nonremovable)
   (e) continuous everywhere
   (f) $x = -2$ (nonremovable), $x = 1$ (removable)

5. The answer is in working the problem.

6. various correct answers

7. (a) v.a.: $x = -3$
   (b) v.a.: $x = 2, x = -2$
   (c) no v.a.

8. (a) -2
   (b) -80 ft/sec

9. (a) 8/35
   (b) 5 m/sec