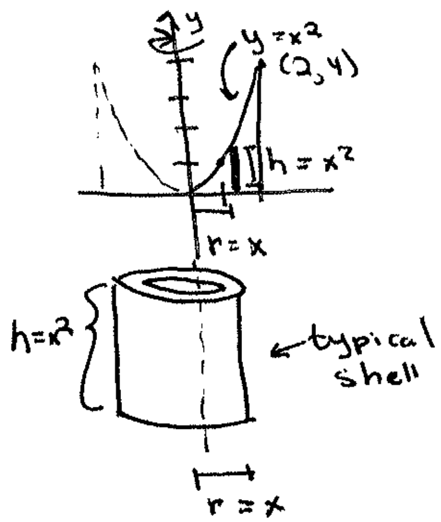
Examples for Section 5.3

For examples 1 and 2 use the region bounded by $y = x^2$, $x = 2$, and $y = 0$.

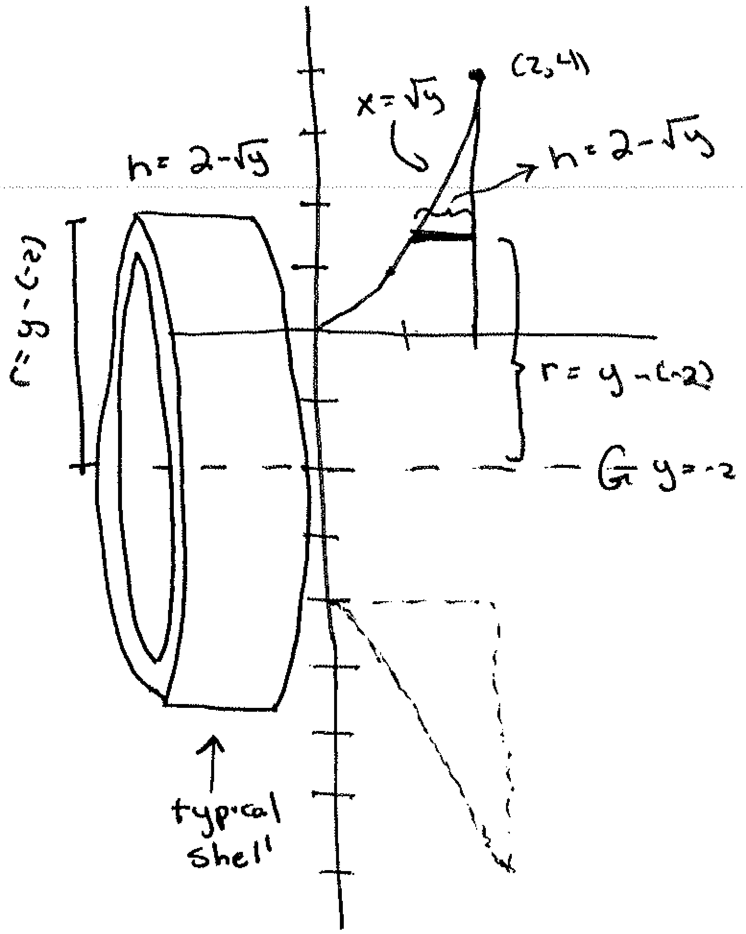
<u>NOTE</u>: Examples 1 and 2 are the same as examples 3 and 2, respectively, in Section 5.2. This is so that you can compare and contrast the Method of Disk/Washers with the Method of Shells

① Find the volume of the solid generated when the region is rotated about the y-axis. Use the Method of Shells.



$$V = 2\pi\int_0^2 x \cdot x^2\,dx$$
$$= 2\pi\int_0^2 x^3\,dx$$
$$= 2\pi \cdot \frac{x^4}{4}\Big]_0^2$$
$$= \pi \cdot \frac{x^4}{2}\Big]_0^2$$
$$= \pi \cdot \frac{2^4}{2} - \pi \cdot 0^4$$
$$= 8\pi$$

② Find the volume of the solid generated by revolving the region about the line $y = -2$. Use the Method of Shells



$$V = 2\pi \int_0^4 [y-(-2)](2-\sqrt{y})\,dy$$

$$= 2\pi \int_0^4 (y+2)(2-y^{1/2})\,dy$$

$$= 2\pi \int_0^4 (2y - y^{3/2} + 4 - 2y^{1/2})\,dy$$

$$= 2\pi \left[y^2 - \tfrac{2}{5} y^{5/2} + 4y - 2\cdot\tfrac{2}{3} y^{3/2} \right]_0^4$$

$$= 2\pi \left[\left(4^2 - \tfrac{2}{5}\cdot 4^{5/2} + 4\cdot 4 - \tfrac{4}{3}\cdot 4^{3/2}\right) - 0 \right]$$

$$= 2\pi \left(16 - \tfrac{64}{5} + 16 - \tfrac{32}{3}\right)$$

$$= 2\pi \left(\tfrac{240}{15} - \tfrac{192}{15} + \tfrac{240}{15} - \tfrac{160}{15}\right)$$

$$= 2\pi \left(\tfrac{128}{15}\right)$$

$$= \frac{256\pi}{15}$$

OBSERVATIONS: In comparing Examples 1 and 2 above with Examples 3 and 2 of Section 5.2, we make the following important observations.

① When using the Method of Disk/Washers to calculate the volume, the slices are perpendicular to axis of rotation.

② When using the Method of Shells to calculate the volume, the slices are parallel to the axis of rotation.

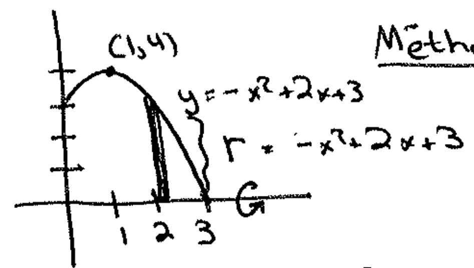
③ Consider the solid generated by revolving the region bounded by $y = -x^2 + 2x + 3$, $y = 0$, and $x = 0$ about the x-axis

(a) Using the Method of Disks, set up an integral to calculate the volume of the solid.

(b) Using the Method of Shells, set up an integral to calculate the volume of the solid.

(c) Evaluate your answer in either (a) or (b) to find the volume of the solid.

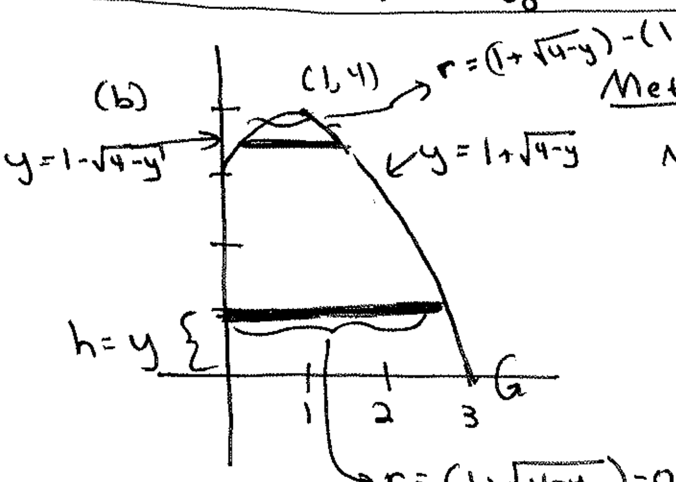
(a)



Method of Disks

$$V = \pi \int_0^3 [-x^2+2x+3]^2 \, dx$$

(b)



Method of Shells

NOTE: Since the slices are parallel to axis of rotation we see that

① we will need horizontal slices and integral will be in terms of y's

② We will need two integrals since the left-hand side changes

Solve: $y = -x^2 + 2x + 3$ for x.

$x^2 - 2x = 3 - y$ ← complete the square

$x^2 - 2x + 1 = 3 - y + 1$

$(x-1)^2 = 4 - y$

$x - 1 = \pm\sqrt{4-y}$ → $x = 1 \pm \sqrt{4-y}$

(over for integral set up)

$$V = 2\pi \int_0^3 \left(1+\sqrt{4-y}\right)\cdot y\,dy + 2\pi\int_3^4 \left[\left(1+\sqrt{4-y}\right)-\left(1-\sqrt{4-y}\right)\right] y\,dy$$

(c) It is easy to see that it will be easier to find the volume using the integral in part (a)

$$V = \pi \int_0^3 (-x^2+2x+3)^2\,dx$$

$$= \pi \int_0^3 (x^4-4x^3-2x^2+12x+9)\,dx$$

$$= \pi \left[\frac{x^5}{5} - x^4 - \frac{2x^3}{3} + 6x^2 + 9x\right]_0^3$$

$$= \pi \left[\left(\frac{3^5}{5} - 3^4 - \frac{2\cdot 3^3}{3} + 6\cdot 3^2 + 9\cdot 3\right) - 0\right]$$

$$= \pi\left(\frac{153}{5}\right) = \frac{153\pi}{5}$$

OBSERVATION: As can be seen in example 3, there is usually one way that is easier to set up the problem than others. Sometimes it is the Method of Disks/Washers, and sometimes it is the Method of Shells. It will depend on the individual problem which one is easier.