Examples for Section 5.2

For examples 1 through 3 use the region bounded by $y = x^2$, $x = 0$, and $x = 2$.

① Find the volume of the solid generated by revolving the region about the x-axis

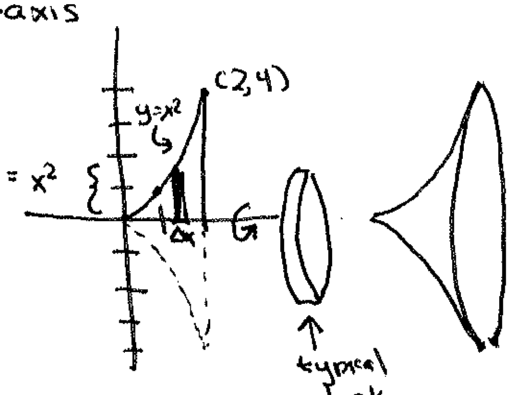
$$V = \int_0^2 \pi (x^2)^2\, dx$$

$$= \int_0^2 \pi x^4 dx$$

$$= \pi \int_0^2 x^4 dx$$

$$= \pi \left[\frac{x^5}{5}\right]_0^2$$

$$= \pi\left[\frac{2^5}{5} - \frac{0^5}{5}\right] = \pi\left[\frac{32}{5} - 0\right] = \frac{32\pi}{5}$$



$\Delta V = \pi (x^2)^2 \Delta x$

② Find the volume of the solid generated by revolving the region about the line $y = -2$.

$$V = \int_0^2 \pi\left[(x^2+2)^2 - 2^2\right] dx$$

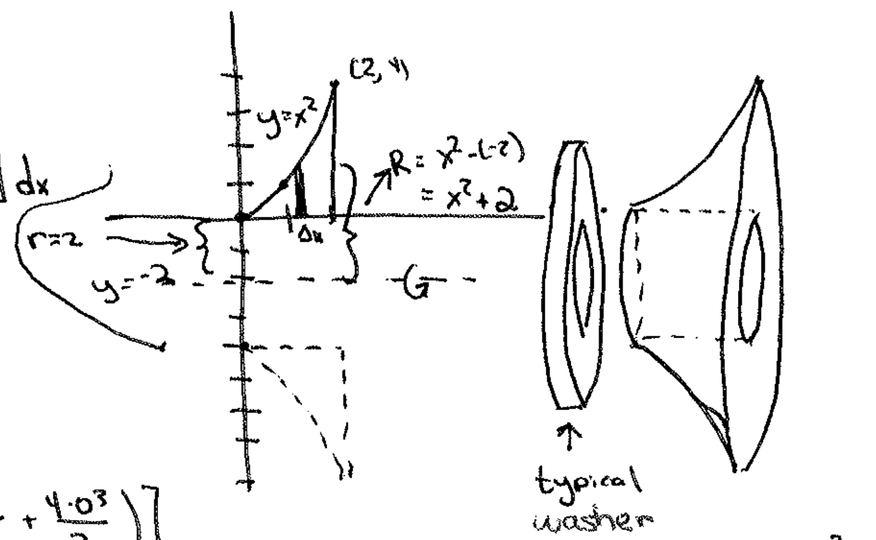
$$= \pi \int_0^2 \left[x^4 + 4x^2 + 4 - 4\right] dx$$

$$= \pi \int_0^2 (x^4 + 4x^2) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{4x^3}{3}\right]_0^2$$

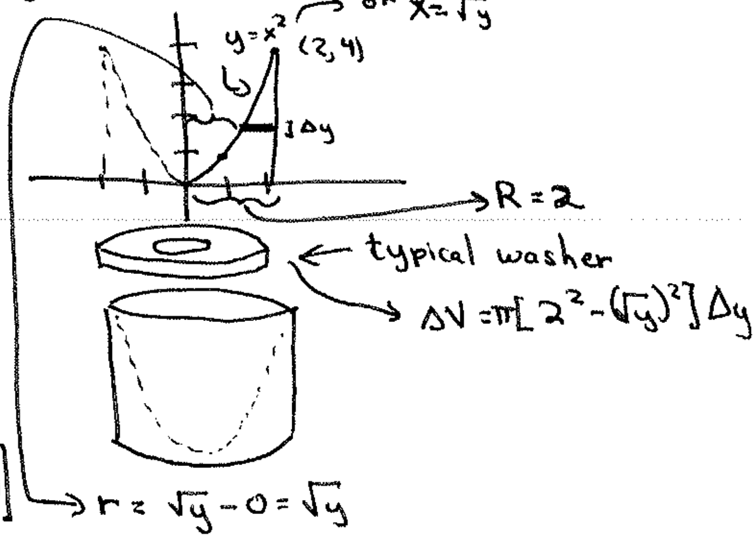
$$= \pi \left[\left(\frac{2^5}{5} + \frac{4\cdot 2^3}{3}\right) - \left(\frac{0^5}{5} + \frac{4\cdot 0^3}{3}\right)\right]$$

$$= \pi\left[\frac{32}{5} + \frac{32}{3}\right] = \pi\left[\frac{96}{15} + \frac{160}{15}\right] = \frac{256\pi}{15}$$



$\Delta V = \pi\left[(x^2+2)^2 - (2)^2\right]\Delta x$

③ Find the volume of the solid generated by revolving the region about the y-axis.



$$V = \int_0^4 \pi[2^2 - (\sqrt{y})^2]\,dy$$

$$= \pi \int_0^4 [4 - y]\,dy$$

$$= \pi \left[4y - \frac{y^2}{2}\right]_0^4$$

$$= \pi \left[\left(4\cdot 4 - \frac{4^2}{2}\right) - \left(4\cdot 0 - \frac{0^2}{2}\right)\right]$$

$$= \pi \left[\left(16 - \frac{16}{2}\right) - 0\right] = \pi(16 - 8) = 8\pi$$

④ Find the volume of the solid generated by revolving the region bound by the line $x - 2y = 0$ and the parabola $y^2 - 2x = 0$ about the line $x = -1$.

First, find the points of intersection of

$x - 2y = 0$ and $y^2 - 2x = 0$

$x = 2y$

$y^2 - 2(2y) = 0$

$y^2 - 4y = 0$

$y(y - 4) = 0$

$y = 0,\ y - 4 = 0$

$y = 4$

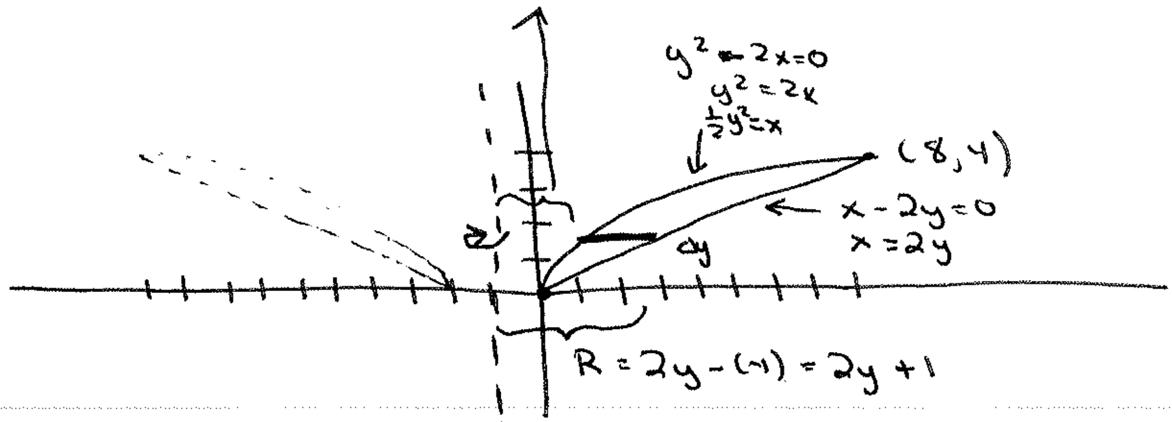
if $y = 0$, $x = 2\cdot 0 = 0$

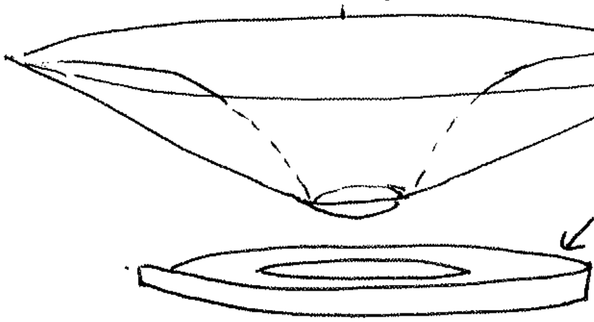
if $y = 4$, $x = 2\cdot 4 = 8$

points of intersection are $(0,0)$ and $(8,4)$

<u>Over</u>

$r = \frac{1}{2}y^2 - (-1) = \frac{1}{2}y^2 + 1$





typical washer

$\Delta V = \pi\left[(2y+1)^2 - (\tfrac{1}{2}y^2+1)^2\right]\Delta y$

$$V = \int_0^4 \pi\left[(2y+1)^2 - (\tfrac{1}{2}y^2+1)^2\right]dy$$

$$= \pi\int_0^4 \left[(4y^2+4y+1) - (\tfrac{1}{4}y^4 + y^2 + 1)\right]dy$$

$$= \pi\int_0^4 (4y^2 + 4y + \cancel{1} - \tfrac{1}{4}y^4 - y^2 - \cancel{1})\,dy$$

$$= \pi\int_0^4 (4y + 3y^2 - \tfrac{1}{4}y^4)\,dy$$

$$= \pi\left[4\frac{y^2}{2} + 3\frac{y^3}{3} - \frac{1}{4}\cdot\frac{y^5}{5}\right]_0^4$$

$$= \pi\left[2y^2 + y^3 - \frac{y^5}{20}\right]_0^4$$

$$= \pi\left[\left(2\cdot 4^2 + 4^3 - \frac{4^5}{20}\right) - \left(2\cdot 0^2 + 0^3 - \frac{0^5}{20}\right)\right]$$

$$= \pi\left[\left(2\cdot 16 + 64 - \frac{1028}{20}\right) - 0\right]$$

$$= \pi\left(32 + 64 - \frac{256}{5}\right)$$

$$= \pi\left(96 - \frac{256}{5}\right) = \pi\left(\frac{480}{5} - \frac{256}{5}\right) = \frac{224\pi}{5}$$