Unit 8B

Doubling Time and Half-Life

DOUBLING TIME AND HALF-LIFE

- The time required for each doubling in exponential growth is called the **doubling time**.
- The time it takes the value of a quantity in exponential decay to decrease to half its value is called the **half-life**.

CALCULATIONS WITH DOUBLING TIME

After a time *t*, an exponentially growing quantity with a doubling time of T_{double} increases in size by a factor of $2^{(t/T_{\text{double}})}$. The new value of the growing quantity is related to its initial value (at $t = 0$) by

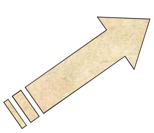
$$\text{new value} = \text{initial value} \times 2^{(t/T_{\text{double}})}$$

CALCULATIONS WITH HALF-LIFE

After time *t*, an exponentially decaying quantity with a half-life of T_{half} increases in size by a factor of $(1/2)^{(t/T_{\text{half}})}$. The new value of the decaying quantity is related to its initial value (at $t = 0$) by

$$\text{new value} = \text{initial value} \times \left(\frac{1}{2}\right)^{(t/T_{\text{half}})}$$

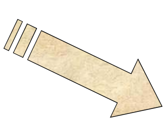
EXPONENTIAL GROWTH VERSUS DECAY



Exponential Growth

$$\text{new value} = \text{initial value} \times 2^{(t/T_{\text{double}})}$$

$$\text{new value} = \text{initial value} \times (1/2)^{(t/T_{\text{half}})}$$



Exponential Decay

APPROXIMATE DOUBLING TIME FORMULA (RULE OF 70)

For a quantity growing exponentially at a rate of *P*% per time period, the doubling time is ***approximately***

$$T_{\text{double}} \approx \frac{70}{P}$$

This approximation works best for small growth rates and breaks down for growth rates over about 15%.

APPROXIMATE HALF-LIFE FORMULA (RULE OF 70)

For a quantity decaying exponentially at a rate of *P*% per time period, the half-life is ***approximately***

$$T_{\text{half}} \approx \frac{70}{P}$$

This approximation works best for small decay rates and breaks down for decay rates over about 15%.

LOGARITHMS

A **logarithm** (or **log**, for short) is a number that represents a power or exponent.

COMMON LOGS

In this course, we will focus on only base 10 logs, also called **common logs**, which are defined as follows

$\log_{10} x$ is the power to which 10 must be raised to obtain *x*.

OR

$\log_{10} x$ means "10 to what power equals *x*?"

PROPERTIES OF LOGS

1. $\log_{10} 10^x = x$
2. $10^{\log_{10} x} = x$
3. $\log_{10}(xy) = \log_{10} x + \log_{10} y$
4. $\log_{10}\left(a^x\right) = x \cdot \log_{10} a$

EXACT DOUBLING TIME AND HALF-LIFE FORMULAS

For an exponentially growing quantity with a ***fractional*** (or ***decimal***) growth rate *r*, the doubling time is

$$T_{\text{double}} = \frac{\log_{10} 2}{\log_{10}(1+r)}$$

For an exponentially decaying quantity, we use a ***negative*** value for *r*. The half-life is

$$T_{\text{half}} = -\frac{\log_{10} 2}{\log_{10}(1+r)}$$